Energy Efficiency of Fixed-Rate Wireless Transmissions under QoS Constraints

Deli Qiao, Mustafa Cenk Gursoy, Senem Velipasalar
Department of Electrical Engineering
University of Nebraska-Lincoln, Lincoln, NE 68588
Email: dqiao726@huskers.unl.edu, gursoy@engr.unl.edu, velipasa@engr.unl.edu

Abstract—[1] **Transmission over wireless fading channels under quality of service (QoS) constraints is studied when only the receiver has perfect channel side information. Being unaware of the channel conditions, transmitter is assumed to send the information at a fixed rate. Under these assumptions, a two-state (ON-OFF) transmission model is adopted, where information is transmitted reliably at a fixed rate in the ON state while no reliable transmission occurs in the OFF state. QoS limitations are imposed as constraints on buffer violation probabilities, and effective capacity formulation is used to identify the maximum arrival rate that a wireless channel can sustain while satisfying statistical QoS constraints. Energy efficiency is investigated by obtaining the minimum bit energy and wideband slope expressions in both low-power and wideband regimes. The increased energy requirements due to the presence of QoS constraints are quantified. Comparisons with variable-rate/fixed-power and variable-rate/variable-power cases are given. Overall, an energy-delay tradeoff for fixed-rate transmission systems is provided.**

I. Introduction

Efficient use of limited energy resources is of paramount importance in most wireless systems. From an information-theoretic perspective, the energy required to reliably send one bit is a metric that can be adopted to measure the energy efficiency. Generally, energy-per-bit requirement is minimized, and hence the energy efficiency is maximized, if the system operates in the low-power or wideband regimes. Recently, Verdú in [1] has determined the minimum bit energy required for reliable communications over a general class of channels, and studied the spectral efficiency–bit energy tradeoff in the wideband regime.

While providing powerful results, information-theoretic studies generally do not address delay and quality of service (QoS) constraints [2]. However, the impact upon the queue length and queueing delay of transmission using codes with large blocklength, which are required to achieve the information-theoretic performance limits, can be significant. Situation is even further exacerbated in wireless channels in which the ergodic capacity has an operational meaning only if the codewords are long enough to span all fading states. Hence, in slow fading environments, large delays can be experienced in order to achieve the ergodic capacity. Due to these considerations, performance metrics such as capacity versus outage [3] and delay limited capacity [4] have been considered in the literature for slow fading scenarios.

[1]This work was supported by the National Science Foundation under Grants CCF – 0546384 (CAREER) and CNS – 0834753.

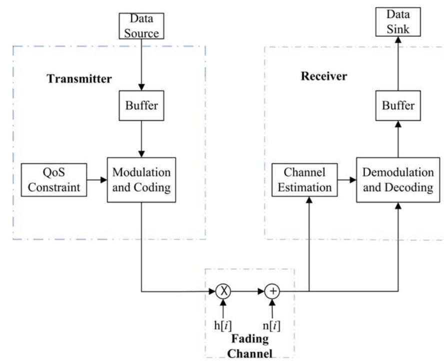

Fig. 1. The general system model.

More recently, Wu and Negi in [5] defined the effective capacity as the maximum constant arrival rate that a given time-varying service process can support while providing statistical QoS guarantees. Effective capacity formulation uses the large deviations theory and incorporates the statistical QoS constraints by capturing the rate of decay of the buffer occupancy probability for large queue lengths. The analysis and application of effective capacity in various settings has attracted much interest recently (see e.g., [6]–[8]).

In this paper, we consider a wireless communication scenario in which only the receiver has the channel side information, and the transmitter, not knowing the channel conditions, sends the information at a fixed-rate. If the fixed-rate transmission cannot be supported by the channel, outage occurs and information has to be retransmitted. In this scenario, we investigate the energy efficiency under QoS constraints in the low-power and wideband regimes by considering the bit energy requirement defined as average energy normalized by the effective capacity.

II. System Model

We consider a point-to-point wireless link in which there is one source and one destination. The system model is depicted in Fig.1. It is assumed that the source generates data sequences which are divided into frames of duration T. These data frames are initially stored in the buffer before they are transmitted over the wireless channel. The discrete-time channel input-output relation in the i^{th} symbol duration is given by

$$y[i] = h[i]x[i] + n[i] \quad i = 1, 2, \ldots. \tag{1}$$

where $x[i]$ and $y[i]$ denote the complex-valued channel input and output, respectively. We assume that the bandwidth available in the system is B and the channel input is subject to the following average energy constraint: $\mathbb{E}\{|x[i]|^2\} \leq \bar{P}/B$ for all i. Since the bandwidth is B, symbol rate is assumed to be B complex symbols per second, indicating that the average power of the system is constrained by $\bar{P}$. Above in (1), $n[i]$ is a zero-mean, circularly symmetric, complex Gaussian random variable with variance $\mathbb{E}\{|n[i]|^2\} = N_0$. The additive Gaussian noise samples $\{n[i]\}$ are assumed to form an independent and identically distributed (i.i.d.) sequence. Finally, $h[i]$ denotes the channel fading coefficient, and $\{h[i]\}$ is a stationary and ergodic discrete-time process. We denote the magnitude-square of the fading coefficients by $z[i] = |h[i]|^2$.

We assume that while the receiver has perfect channel side information and hence perfectly knows the instantaneous values of $\{h[i]\}$, the transmitter has no such knowledge. Under this assumption, the instantaneous capacity of the channel with channel gain $z[i] = |h[i]|^2$ is

$$C[i] = B \log_2(1 + \text{SNR} z[i]) \text{ bits/s} \tag{2}$$

where $\text{SNR} = \bar{P}/(N_0 B)$ is the average transmitted signal-to-noise ratio. Since the transmitter is unaware of the channel conditions, information is transmitted at a fixed rate of r bits/s. When $r < C$, the channel is considered to be in the ON state and reliable communication is achieved at this rate. If, on the other hand, $r \geq C$, outage occurs. In this case, channel is in the OFF state and reliable communication at the rate of r bits/s cannot be attained. Hence, effective data rate is zero and information has to be resent. We assume that a simple automatic repeat request (ARQ) mechanism is incorporated in the communication protocol to acknowledge the reception of data and to ensure that the erroneous data is retransmitted [6].

Fig. 2 depicts the two-state transmission model together with the transition probabilities. In this paper, we assume that the channel fading coefficients stay constant over the frame duration T. Hence, the state transitions occur at every T seconds. Now, the probability of staying in the ON state, i.e., p_{22}, is defined as follows[2]:

$$\begin{aligned} p_{22} &= P\{r < C[i + TB] \,|\, r < C[i]\} \\ &= P\{z[i + TB] > \alpha \,|\, z[i] > \alpha\} \end{aligned} \tag{3}$$

where

$$\alpha = \frac{2^{\frac{r}{B}} - 1}{\text{SNR}}. \tag{4}$$

Note that p_{22} depends on the joint distribution of $(z[i + TB], z[i])$. For the Rayleigh fading channel, the joint density function of the fading amplitudes can be obtained in closed-form [11]. In this paper, in order to simplify the analysis while considering general fading distributions, we assume that

[2]The formulation in (3) assumes as before that the symbol rate is B symbols/s and hence we have TB symbols in a duration of T seconds.

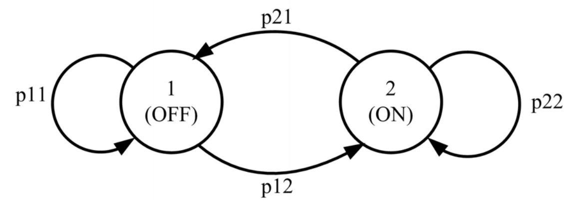

Fig. 2. ON-OFF state transition model.

fading realizations are independent for each frame. Hence, we basically consider a block-fading channel model. Note that in block-fading channels, the duration T over which the fading coefficients stay constant can be used as a parameter to model how fast or slow the fading varies.

Under the block fading assumption, we now have $p_{22} = P\{z[i + TB] > \alpha\} = P\{z > \alpha\}$. Similarly, the other transition probabilities become

$$p_{11} = p_{21} = P\{z \leq \alpha\} = \int_0^{\alpha} p_z(z) dz \tag{5}$$

$$p_{22} = p_{12} = P\{z > \alpha\} = \int_{\alpha}^{\infty} p_z(z) dz \tag{6}$$

where p_z is the density function of z. We finally note that rT bits are successfully transmitted and received in the ON state, while the effective transmission rate in the OFF state is zero.

III. Preliminaries – Effective Capacity and Spectral Efficiency-Bit Energy Tradeoff

In [5], Wu and Negi defined the effective capacity as the maximum constant arrival rate[3] that a given service process can support in order to guarantee a statistical QoS requirement specified by the QoS exponent θ. If we define Q as the stationary queue length, then θ is the decay rate of the tail distribution of the queue length Q:

$$\lim_{q \to \infty} \frac{\log P(Q \geq q)}{q} = -\theta. \tag{7}$$

Therefore, for large $q_{\max}$, we have the following approximation for the buffer violation probability: $P(Q \geq q_{\max}) \approx e^{-\theta q_{max}}$. Hence, while larger θ corresponds to more strict QoS constraints, smaller θ implies looser QoS guarantees. Similarly, if D denotes the steady-state delay experienced in the buffer, then $P(D \geq d_{\max}) \approx e^{-\theta \delta d_{\max}}$ for large $d_{\max}$, where δ is determined by the arrival and service processes [7].

The effective capacity is given by

$$-\frac{\Lambda(-\theta)}{\theta} = -\lim_{t \to \infty} \frac{1}{\theta t} \log_e \mathbb{E}\{e^{-\theta S[t]}\} \tag{8}$$

where $S[t] = \sum_{k=1}^{t} R[k]$ is the time-accumulated service process and $\{R[k], k = 1, 2, \ldots\}$ denote the discrete-time stationary and ergodic stochastic service process. Note that in the model we consider, $R[k] = rT$ or 0 depending on the

[3]For time-varying arrival rates, effective capacity specifies the effective bandwidth of the arrival process that can be supported by the channel.

channel state being ON or OFF, respectively. In [9] and [10], it is shown that for such an ON-OFF model, we have

$$\frac{\Lambda(\theta)}{\theta} = \frac{1}{\theta}\log_e\Big(\frac{1}{2}\Big(p_{11} + p_{22}e^{\theta T r} + \sqrt{(p_{11} + p_{22}e^{\theta T r})^2 + 4(p_{11} + p_{22} - 1)e^{\theta T r}}\Big)\Big). \tag{9}$$

Note that $p_{11} + p_{22} = 1$ in our model. Then, for a given QoS delay constraint θ, the effective capacity normalized by the frame duration T and bandwidth B, or equivalently spectral efficiency in bits/s/Hz, becomes

$$\begin{aligned}
\mathsf{R}_E(\text{SNR}, \theta) &= \frac{1}{TB}\max_{r\geq 0}\Big\{-\frac{\Lambda(-\theta)}{\theta}\Big\} \\
&= \max_{r\geq 0}\Big\{-\frac{1}{\theta TB}\log_e\big(p_{11} + p_{22}e^{-\theta T r}\big)\Big\} \qquad (10)\\
&= \max_{r\geq 0}\Big\{-\frac{1}{\theta TB}\log_e\big(1 - P\{z > \alpha\}(1 - e^{-\theta T r})\big)\Big\} \qquad (11)\\
&= -\frac{1}{\theta TB}\log_e\Big(1 - P\{z > \alpha_{\text{opt}}\}\big(1 - e^{-\theta T r_{\text{opt}}}\big)\Big) \\
&\qquad \text{bits/s/Hz} \qquad (12)
\end{aligned}$$

where r_{opt} is the maximum fixed transmission rate that solves (11) and $\alpha_{\text{opt}} = (2^{\frac{r_{\text{opt}}}{B}} - 1)/\text{SNR}$. Note that both α_{opt} and r_{opt} are functions of SNR and θ.

The normalized effective capacity, R_E, provides the maximum throughput under statistical QoS constraints in the fixed-rate transmission model. It can be easily shown that

$$\lim_{\theta\to 0}\mathsf{R}_E(\text{SNR}, \theta) = \max_{r\geq 0}\ \frac{r}{B}P\{z > \alpha\}. \tag{13}$$

Hence, as the QoS requirements relax, the maximum constant arrival rate approaches the average transmission rate. On the other hand, for $\theta > 0$, $\mathsf{R}_E < \frac{1}{B}\max_{r\geq 0} rP\{z > \alpha\}$ in order to avoid violations of QoS constraints.

In this paper, we focus on the energy efficiency of wireless transmissions under the aforementioned statistical QoS limitations. Since energy efficient operation generally requires operation at low-SNR levels, our analysis throughout the paper is carried out in the low-SNR regime. In this regime, the tradeoff between the normalized effective capacity (i.e, spectral efficiency) R_E and bit energy $\frac{E_b}{N_0} = \frac{\text{SNR}}{\mathsf{R}_E(\text{SNR})}$ is a key tradeoff in understanding the energy efficiency, and is characterized by the bit energy at zero spectral efficiency and wideband slope provided, respectively, by

$$\frac{E_b}{N_0}\bigg|_{\mathsf{R}=0} = \lim_{\text{SNR}\to 0}\frac{\text{SNR}}{\mathsf{R}_E(\text{SNR})} = \frac{1}{\dot{\mathsf{R}}_E(0)} \quad \text{and} \tag{14}$$

$$\mathcal{S}_0 = -\frac{2(\dot{\mathsf{R}}_E(0))^2}{\ddot{\mathsf{R}}_E(0)}\log_e 2 \tag{15}$$

where $\dot{\mathsf{R}}_E(0)$ and $\ddot{\mathsf{R}}_E(0)$ are the first and second derivatives with respect to SNR, respectively, of the function $\mathsf{R}_E(\text{SNR})$ at zero SNR [1].

IV. Low-Power Regime

In this section, we investigate the spectral efficiency–bit energy tradeoff as the average power $\bar{P}$ diminishes. In this regime, $\text{SNR} = \bar{P}/(N_0 B)$ vanishes with decreasing $\bar{P}$. Note that we assume that the bandwidth allocated to the channel is fixed. The following result provides the expressions for the bit energy at zero spectral efficiency and the wideband slope.

Theorem 1: In the low-power regime, the bit energy at zero spectral efficiency and wideband slope are given by

$$\frac{E_b}{N_0}\bigg|_{\mathsf{R}=0} = \frac{\log_e 2}{\alpha^*_{\text{opt}}P\{z > \alpha^*_{\text{opt}}\}} \quad \text{and} \tag{16}$$

$$\mathcal{S}_0 = \frac{2P\{z > \alpha^*_{\text{opt}}\}}{1 + \beta(1 - P\{z > \alpha^*_{\text{opt}}\})}, \tag{17}$$

respectively, where $\beta = \frac{\theta T B}{\log_e 2}$ is normalized QoS constraint. In the above formulation, α^*_{opt} is defined as $\alpha^*_{\text{opt}} = \lim_{\text{SNR}\to 0}\alpha_{\text{opt}}$, and α^*_{opt} satisfies

$$\alpha^*_{\text{opt}}p_z(\alpha^*_{\text{opt}}) = P\{z > \alpha^*_{\text{opt}}\}. \tag{18}$$

Proof: We first consider the Taylor series expansion of r_{opt} in the low-SNR regime:

$$r_{\text{opt}} = a\text{SNR} + b\text{SNR}^2 + o(\text{SNR}^2) \tag{19}$$

where a and b are real-valued constants. Substituting (19) into (4), we obtain the Taylor series expansion for α_{opt}:

$$\alpha_{\text{opt}} = \frac{a\log_e 2}{B} + \Big(\frac{b\log_e 2}{B} + \frac{a^2\log_e^2 2}{2B^2}\Big)\text{SNR} + o(\text{SNR}). \tag{20}$$

From (20), we note that in the limit as $\text{SNR}\to 0$, we have

$$\alpha^*_{\text{opt}} = \frac{a\log_e 2}{B}. \tag{21}$$

Next, we obtain the Taylor series expansion with respect to SNR for $P\{z > \alpha_{\text{opt}}\}$ using the Leibniz Integral Rule:

$$P\{z > \alpha_{\text{opt}}\} = P\{z > \alpha^*_{\text{opt}}\} - \Big(\frac{b\log_e 2}{B} + \frac{a^2\log_e^2 2}{2B^2}\Big)p_z(\alpha^*_{\text{opt}})\text{SNR} + o(\text{SNR}). \tag{22}$$

Using (19), (20), and (22), we find the following series expansion for R_E:

$$\begin{aligned}
\mathsf{R}_E(\text{SNR}) &= -\frac{1}{\theta TB}\log_e\Bigg[1 - \Bigg(P\{z > \alpha^*_{\text{opt}}\} - \Big(\frac{b\log_e 2}{B} + \frac{a^2\log_e^2 2}{2B^2}\Big)p_z(\alpha^*_{\text{opt}})\text{SNR} + o(\text{SNR})\Bigg) \\
&\qquad \times \Big(\theta Ta\text{SNR} + (\theta Tb - \frac{(\theta Ta)^2}{2})\text{SNR}^2 + o(\text{SNR}^2)\Big)\Bigg] \\
&= \frac{aP\{z > \alpha^*_{\text{opt}}\}}{B}\text{SNR} + \frac{1}{B}\Big(-\frac{\theta Ta^2}{2}P\{z > \alpha^*_{\text{opt}}\} - \frac{a^3 p_z(\alpha^*_{\text{opt}})\log_e^2 2}{2B^2} + \frac{\theta T(P\{z > \alpha^*_{\text{opt}}\}a)^2}{2}\Big)\text{SNR}^2 \\
&\qquad + o(\text{SNR}^2).
\end{aligned} \tag{23}$$

Then, using (21), we immediately derive from (23) that

$$\dot{\mathsf{R}}_E(0) = \frac{\alpha_{\text{opt}}^* P\{z > \alpha_{\text{opt}}^*\}}{\log_e 2} \quad (24)$$

$$\ddot{\mathsf{R}}_E(0) = -\frac{{\alpha_{\text{opt}}^*}^3 p_z\{\alpha_{\text{opt}}^*\}}{\log_e 2} - \frac{\theta T B {\alpha_{\text{opt}}^*}^2}{\log_e^2 2} P\{z > \alpha_{\text{opt}}^*\}(1 - P\{z > \alpha_{\text{opt}}^*\}). \quad (25)$$

Next,we derive an equality satisfied by α_{opt}^*. Consider the objective function in (11):

$$-\frac{1}{\theta TB} \log_e \left(1 - P\{z > \alpha\}(1 - e^{-\theta T r})\right) \quad (26)$$

It can easily be seen that both as $r \to 0$ and $r \to \infty$, this objective function approaches zero[4]. Hence, (26) is maximized at a finite and nonzero value of r at which the derivative of (26) with respect to r is zero. Differentiating (26) with respect to r and making it equal to zero leads to the following equality that needs to be satisfied at the optimal value r_{opt}:

$$\frac{2^{r_{\text{opt}}/B} p_z(\alpha_{\text{opt}}) \log_e 2}{B\text{SNR}} (1 - e^{-\theta T r_{\text{opt}}}) = \theta T e^{-\theta T r_{\text{opt}}} P\{z > \alpha_{\text{opt}}\}. \quad (27)$$

Taking the limits of both sides of (27) as $\text{SNR} \to 0$ and employing (19), we obtain

$$\frac{a p_z(\alpha_{\text{opt}}^*) \log_e 2}{B} = P\{z > \alpha_{\text{opt}}^*\}. \quad (28)$$

From (21), (28) simplifies to

$$\alpha_{\text{opt}}^* p_z(\alpha_{\text{opt}}^*) = P\{z > \alpha_{\text{opt}}^*\}. \quad (29)$$

This proves the equality condition stated in the Theorem. Moreover, using (29), the first term in the expression for $\ddot{\mathsf{R}}_E(0)$ in (25) becomes $-\frac{{\alpha_{\text{opt}}^*}^2 P\{z > \alpha_{\text{opt}}^*\}}{\log_e 2}$. Together with this change, evaluating the expressions in (14) with the results in (24) and (25), we obtain (16) and (17). □

The following result shows that $\frac{E_b}{N_0}\big|_{\mathsf{R}=0}$ is the minimum bit energy when the magnitude-square of the fading coefficients, z, is Gamma distributed. Note that the distribution of z in Nakagami-m and Rayleigh fading channels can be obtained as special cases of the Gamma distribution.

Theorem 2: If z is Gamma distributed and hence the probability density function of z is given by

$$p_z(z) = \frac{\lambda^\beta}{\Gamma(\beta)} z^{\beta-1} e^{-\lambda z} \quad (30)$$

where $\beta \geq 1$ and $\lambda > 0$, then the bit energy required at zero spectral efficiency is indeed the minimum one, i.e. $\frac{E_b}{N_0}\big|_{\mathsf{R}=0} = \frac{E_b}{N_0}_{\text{min}}$, for all $\theta \geq 0$.

Next, we provide numerical results. Throughout the paper, we set the frame duration to $T = 2$ms. For the fixed bandwidth case, we have assumed $B = 10^5$ Hz. Fig. 3 plots the spectral efficiency as a function of the bit energy for different values of θ in Nakagami-m channel with $\mathbb{E}\{|h|^2\} = \mathbb{E}\{z\} = 1$ and $m = 1$. The *pdf* of z in Nakagami-m channels is [12]

$$p_z(z) = \frac{m^m z^{m-1}}{\Gamma(m)} e^{-mz} \quad (31)$$

where $\Gamma(\cdot)$ is the Gamma function. Note that when $m = 1$, we have the Rayleigh fading channel. In all cases in Fig. 3, we readily note that $\frac{E_b}{N_0}\big|_{\mathsf{R}=0} = \frac{E_b}{N_0}_{\text{min}}$. Moreover, the minimum bit energy is the same and is equal to the one achieved when there are no QoS constraints (i.e., when $\theta = 0$). From the equation $\alpha_{\text{opt}}^* p_z(\alpha_{\text{opt}}^*) = P\{z > \alpha_{\text{opt}}^*\}$, we can find that $\alpha_{\text{opt}}^* = 1$ in the Rayleigh channel for which $p_z(\alpha_{\text{opt}}^*) = P\{z > \alpha_{\text{opt}}^*\} = e^{-\alpha_{\text{opt}}^*}$. Hence, the minimum bit energy is $\frac{E_b}{N_0}_{\text{min}} = 2.75$ dB. On the other hand, the wideband slopes are $\mathcal{S}_0 = \{0.7358, 0.6223, 0.2605, 0.0382, 0.0040\}$ for $\theta = \{0, 0.001, 0.01, 0.1, 1\}$, respectively. Hence, $\mathcal{S}_0$ decreases with increasing θ and consequently more bit energy is required at a fixed nonzero spectral efficiency.

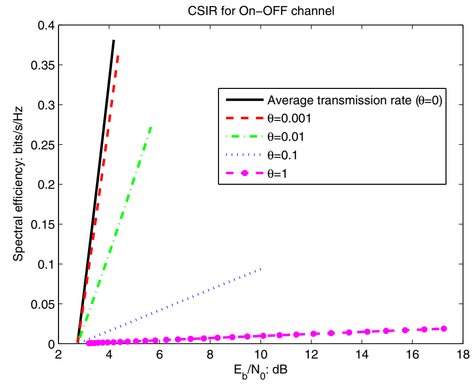

Fig. 3. Spectral efficiency vs. E_b/N_0 in the Rayleigh channel (equivalently Nakagami-m channel with $m = 1$).

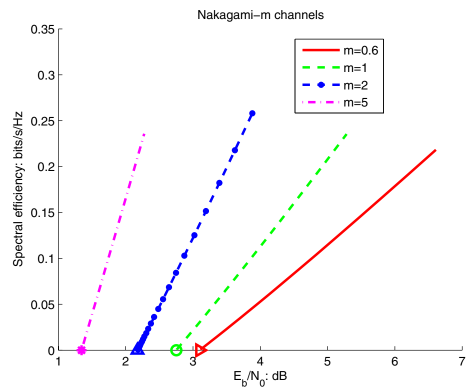

Fig. 4. Spectral efficiency vs. E_b/N_0 in Nakagami-m channels; $\theta = 0.01$, $m = 0.6, 1, 2, 5$.

Fig. 4 plots the spectral efficiency curves as a function of the bit energy for Nakagami-m channels with different m values. θ is set to be 0.01. For $m = \{0.6, 1, 2, 5\}$, we compute that $\alpha_{\text{opt}}^* = \{1.2764, 1, 0.809, 0.7279\}$, $\frac{E_b}{N_0}_{\text{min}} = \{3.099, 2.751, 2.176, 1.343\}$, and $\mathcal{S}_0 =$

[4]Note that α increases without bound with increasing r.

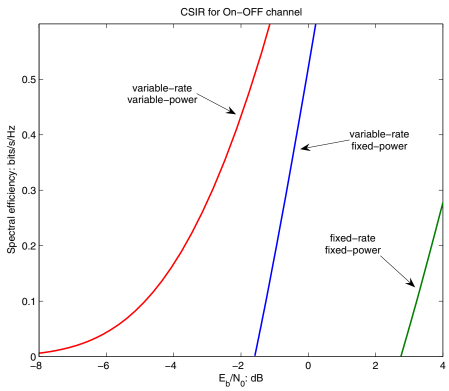

Fig. 5. Spectral efficiency vs. E_b/N_0 in the Rayleigh channel; $\theta = 0.001$.

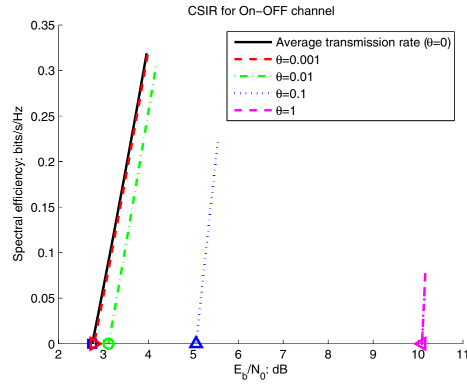

Fig. 6. Spectral efficiency vs. E_b/N_0 in the Rayleigh channel.

$\{0.1707, 0.2605, 0.4349, 0.7479\}$, respectively. We observe that as m increases and hence the channel quality improves, lower bit energies are required. Finally, in Fig. 5, we plot the spectral efficiency vs. E_b/N_0 for different transmission strategies. The variable-rate/variable-power and variable-rate/fixed-power strategies are studied in [8]. We immediately see that substantially more energy is required for fixed-rate/fixed-power transmission schemes considered in this paper.

V. Wideband Regime

In this section, we consider the wideband regime in which the bandwidth is large. We assume that the average power $\bar{P}$ is kept constant. Note that as the bandwidth B increases, $\text{SNR} = \frac{\bar{P}}{N_0 B}$ approaches zero and we again operate in the low-SNR regime similarly as in Section IV. However, energy requirements in the wideband regime will be different from those in the low-power regime as will be evident with the result of Theorem 3.

We introduce the notation $\zeta = \frac{1}{B}$. Note that as $B \to \infty$, we have $\zeta \to 0$. Moreover, with this notation, the effective capacity can be expressed as

$$\mathsf{R}_E(\text{SNR}) = -\frac{\zeta}{\theta T} \log_e \left(1 - P\{z > \alpha_{\text{opt}}\}(1 - e^{-\theta T r_{\text{opt}}})\right). \quad (32)$$

Note that α_{opt} and r_{opt} are also in general dependent on B and hence ζ. The following result provides the minimum bit energy and wideband slope expressions in the wideband regime.

Theorem 3: In the wideband regime, $\frac{E_b}{N_0}_{\text{min}}$ and wideband slope are given by

$$\frac{E_b}{N_0}_{\text{min}} = \frac{-\delta \log_e 2}{\log_e \xi} \quad \text{and} \quad (33)$$

$$\mathcal{S}_0 = \frac{2\xi \log_e^2 \xi}{(\delta\alpha^*_{\text{opt}})^2 P\{z > \alpha^*_{\text{opt}}\} e^{-\delta\alpha^*_{\text{opt}}}}, \quad (34)$$

respectively, where $\delta = \frac{\theta T \bar{P}}{N_0 \log_e 2}$ and $\xi = 1 - P\{z > \alpha^*_{\text{opt}}\}(1 - e^{-\delta\alpha^*_{\text{opt}}})$. Still, α^*_{opt} is defined as $\alpha^*_{\text{opt}} = \lim_{\zeta \to 0} \alpha_{\text{opt}}$ and α^*_{opt} satisfies

$$\delta\alpha^*_{\text{opt}} = \log_e\left(1 + \delta \frac{P\{z > \alpha^*_{\text{opt}}\}}{p_z(\alpha^*_{\text{opt}})}\right). \quad (35)$$

Unlike in the low-power regime, the minimum bit energy in the wideband regime depends on θ . Fig. 6 plots the spectral efficiency curves as a function of bit energy in the Rayleigh channel. In all the curves, we set $\bar{P}/N_0 = 10^4$. We compute that $\alpha^*_{\text{opt}} = \{1, 0.9858, 0.8786, 0.4704, 0.1177\}$ from which we obtain $\frac{E_b}{N_0}_{\text{min}} = \{2.75, 2.79, 3.114, 5.061, 10.087\}$dB for $\theta = \{0, 0.001, 0.01, 0.1, 1\}$, respectively. For the same set of θ values in the same sequence, we compute the wideband slope values as $\mathcal{S}_0 = \{0.7358, 0.7463, 0.8345, 1.4073, 3.1509\}$. We immediately observe that more stringent QoS constraints and hence higher values of θ lead to higher minimum bit energy values and also higher energy requirements at other nonzero spectral efficiencies.

References

[1] S. Verdú, "Spectral efficiency in the wideband regime," *IEEE Trans. Inform. Theory*, vol.48, no.6 pp.1319-1343. Jun. 2002.

[2] A. Ephremides and B. Hajek, "Information theory and communication networks: An unconsummated union," *IEEE Trans. Inform. Theory*, vol. 44, pp. 2416-2434, Oct. 1998.

[3] L. Ozarow, S. Shamai (Shitz), and A. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Trans. Veh. Technol.*, vol. 43, pp. 359-378, May 1994.

[4] S. V. Hanly and D.N.C Tse, "Multiaccess fading channels-part II: delay-limited capacities," *IEEE Trans. Inform. Theory*, vol.44, no.7, pp.2816-2831. Nov. 1998.

[5] D. Wu and R. Negi "Effective capacity: a wireless link model for support of quality of service," *IEEE Trans. Wireless Commun.*, vol.2,no. 4, pp.630-643. July 2003

[6] L. Liu, P. Parag, J. Tang, W.-Y. Chen and J.-F. Chamberland, "Resource allocation and quality of service evaluation for wireless communication systems using fluid models," *IEEE Trans. Inform. Theory*, vol. 53, no. 5, pp. 1767-1777, May 2007

[7] J. Tang and X. Zhang, "Cross-layer-model based adaptive resource allocation for statistical QoS guarantees in mobile wireless networks," *IEEE Trans. Wireless Commun.*, vol. 7, pp.2318-2328, June 2008.

[8] D. Qiao, M.C. Gursoy and S. Velipasalar, "Analysis of energy efficiency in fading channel under QoS constrains," *IEEE Global Communications Conference (GLOBECOM)*, New Orleans, Dec. 2008.

[9] C.-S. Chang, "Stability, queue length, and delay of deterministic and stochastic queuing networks," *IEEE Trans. Auto. Control*, vol. 39, no. 5, pp. 913-931, May 1994

[10] C.-S. Chang, *Performance Guarantees in Communication Networks*, New York: Springer, 1995

[11] P. Sadeghi and P. Rapajic, "Capacity analysis for finite-state Markov mapping of flat-fading channels," *IEEE Tran. Commun.* vol. 53, pp. 833-840, May 2005.

[12] A. Goldsmith, *Wireless Communications*, 1st ed. Cambridge University Press, 2005.