

Performance Analysis for Multichannel Reception of OOFSK Signaling

Qingyun Wang Mustafa Cenk Gursoy
 Department of Electrical Engineering
 University of Nebraska-Lincoln, Lincoln, NE 68588
 Email: qwang4@bigred.unl.edu, gursoy@unl.edu

Abstract—¹ In this paper, the error performance of on-off frequency shift keying (OOFSK) modulation over fading channels is analyzed when the receiver is equipped with multiple antennas. The analysis is conducted in two cases: the coherent scenario where the fading is perfectly known at the receiver, and the noncoherent scenario where neither the receiver nor the transmitter knows the fading coefficients. For both cases, the maximum a posteriori probability (MAP) detection rule is derived and analytical probability of error expressions are obtained. The effect of fading correlation among the receiver antennas is also studied. Simulation results indicate that for sufficiently low duty cycle values, lower probability of error values with respect to FSK signaling are achieved. Equivalently, when compared to FSK modulation, OOFSK with low duty cycle requires less energy to achieve the same probability of error, which renders this modulation a more energy efficient transmission technique.

I. INTRODUCTION

Frequency-shift keying (FSK) is a modulation format that is well-known and well-studied in the communications literature [15]. FSK is an attractive transmission scheme due to its high energy efficiency and suitability for noncoherent communications. In unknown channel conditions, energy detection can be employed to detect the FSK signals. Indeed, the analysis of FSK modulation dates back to 1960s (see e.g., [1], and [2]). Recently, it has been shown in [5] that unless the channel conditions are perfectly known at the receiver, signals that have very high peak-to-average power ratio is required to achieve the capacity in the low SNR regime. This has initiated work on peaky signaling. Luo and Médard [6] have shown that FSK with small duty cycle can achieve rates of the order of capacity in ultrawideband systems with limits on bandwidth and peak power. In [8], the authors have studied the error performance of peaky FSK signaling over multipath fading channels by obtaining upper and lower bounds on the error probability. In [9], on-off frequency-shift keying (OOFSK) is defined as FSK overlaid on on-off keying, and its capacity and energy efficiency is analyzed. Note that OOFSK can be seen as joint pulse position modulation (PPM) and FSK. In this signaling, peakedness is introduced in both time and frequency. The error performance of OOFSK signaling when the transmitter and receiver are each equipped with a single antenna is recently studied in [10].

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One of the important techniques to improve the performance in wireless communications is to use multiple antennas to achieve diversity gain. Considerable amount of work has been done on multiple reception channels. In [2], it is shown for binary and M -ary signaling over Rician fading channels that increasing the number of reception channels can improve the error performance significantly. By finding the probability distribution function of the instantaneous SNR in flat fading multi-reception channels and substituting it into the probability of error expressions of PAM, PSK and QAM over AWGN channel, the authors in [12] obtained expressions for the average probability of error of multi-reception fading channels. In [14], the probability of error of BPSK over Rician fading multi-reception channels is given and extensions to other modulation techniques are discussed. In [17], average symbol error rate of selection diversity of M -ary FSK modulated signal transmitted over fading channels is studied.

In this paper, the error performance of OOFSK over multiple reception Rician fading channels is studied. In Section II, the system model is presented. In Section III, the error performance in coherent fading channels is studied. In Section IV, we investigate the error performance in noncoherent Rician fading channels.

II. SYSTEM MODEL

We assume that OOFSK modulation is employed at the transmitter to send the information. In OOFSK modulation, the transmitted signal during the symbol interval $0 \leq t \leq T_s$ can be expressed as

$$s_m(t) = \begin{cases} \sqrt{\frac{P}{v}} e^{j(w_m t + \theta_m)} & m = 1, 2, 3, \dots, M \\ 0 & m = 0 \end{cases} \quad (1)$$

where w_m and θ_m are the frequency in radians per second and phase, respectively, of the signal $s_m(t)$ when $m \neq 0$. Note that we have M FSK signals and a zero signal denoted by $s_0(t)$. The frequencies of the FSK signals are chosen so that the signals are orthogonal. It is assumed that an FSK signal $s_m(t)$, $m \neq 0$, is transmitted with a probability of $\frac{v}{M}$ while $s_0(t)$ is transmitted with a probability of $1 - v$ where v is the duty cycle of transmission. With these definitions, it is easily seen that P and $\frac{P}{v}$ are the average and peak powers, respectively, of the modulation technique.

The receiver is equipped with L antennas that enable the multiple reception of the transmitted signal. If, without loss

of generality, we assume that $s_k(t)$ is the transmitted signal, the received signal at the l^{th} antenna is

$$r_l(t) = h_l s_k(t) + n_l(t) \quad l = 1, 2, \dots, L \quad (2)$$

where h_l is the fading coefficient at the l^{th} reception channel and $n_l(t)$ is a white Gaussian noise with single-sided spectral density of N_0 . It is assumed that the additive Gaussian noise components at different antennas are independent. Furthermore, the received signal model (2) presumes that the fading is frequency-flat and slow enough so that the fading coefficient stays constant over one symbol duration.

Following each antenna, there is a bank of M correlators, each correlating the received signal with one of the orthogonal frequencies. The output of the m^{th} correlator employed after the l^{th} antenna is given by

$$\begin{aligned} Y_{l,m} &= \frac{1}{\sqrt{N_0 T_s}} \int_0^{T_s} r_l(t) e^{-j\omega_m t} dt \\ &= \begin{cases} \sqrt{\frac{P T_s}{v N_0}} h_l e^{j\theta_m} + n_{l,m} & m = k \\ n_{l,m} & m \neq k \end{cases} \\ &= \begin{cases} A h_l e^{j\theta_m} + n_{l,m} & m = k \\ n_{l,m} & m \neq k, \end{cases} \end{aligned} \quad (3)$$

where $n_{l,m}$ is a circularly symmetric complex Gaussian random variable with zero-mean and a variance of $\frac{1}{v}$ and for notational convenience, we have defined $A = \sqrt{\frac{P T_s}{v N_0}}$. Since the frequencies are orthogonal and the additive Gaussian noise is independent at each antenna, $\{n_{l,m}\}$ for $l \in \{1, \dots, L\}$ and $m \in \{1, \dots, M\}$ forms an independent and identically distributed (i.i.d.) sequence. Note also that $R_{l,m} = |Y_{l,m}|^2$ gives the energy present in the m^{th} frequency at the l^{th} antenna.

III. OOFSK OVER COHERENT FADING CHANNELS

A. Detection Rule

In this section, we assume that transmission takes place over coherent fading channels and hence h_l for all l is known to the receiver while the transmitter does not have such knowledge. Conditioned on h_l and the transmitted signal $s_k(t)$, $Y_{l,m}$ is a proper complex Gaussian random variable with mean value and variance given by

$$E\{Y_{l,m}|h_l, s_k\} = \begin{cases} A h_l e^{j\theta_k} & m = k \\ 0 & m \neq k \end{cases} \quad (4)$$

$$\text{var}\{Y_{l,m}|h_l, s_k\} = 1. \quad (5)$$

Therefore, $R_{l,m} = |Y_{l,m}|^2$ is chi-square distributed with the following conditional probability density function (pdf):

$$f_{R_{l,m}|h_l, s_k}(R_{l,m}) = \begin{cases} e^{-(R_{l,m} + A^2 |h_l|^2)} I_0(2A|h_l|\sqrt{R_{l,m}}) & m = k \\ e^{-R_{l,m}} & m \neq k \end{cases}$$

It is assumed that the receiver, using equal gain combining (EGC), combines the energies of the m^{th} frequency components at each antenna, i.e., computes the total energy

$$R_m = \sum_{l=1}^L R_{l,m}. \quad (6)$$

Since the noise components are independent, R_m is a sum of independent chi-square random variables, and is itself also chi-square distributed with $2L$ degrees of freedom. The conditional pdf is given by

$$f_{R_m|\mathbf{h}, s_k}(R_m) = \begin{cases} \left(\frac{R_m}{\xi}\right)^{\frac{L-1}{2}} e^{-(R_m + \xi)} I_{L-1}(2\sqrt{R_m \xi}) & m = k \\ \frac{R_m^{L-1}}{\Gamma(L)} e^{-R_m} & m \neq k \end{cases}$$

where $\xi = \sum_{l=1}^L A^2 |h_l|^2$, $\mathbf{h} = [h_1, \dots, h_L]$, $I_{L-1}(\cdot)$ is the $(L-1)^{\text{th}}$ order modified Bessel function of the first kind, and $\Gamma(\cdot)$ is the gamma function.

The receiver employs maximum a posteriori probability (MAP) criterion to detect the transmitted signals. Let

$$\mathbf{R} = [R_1, R_2, \dots, R_M]$$

be the vector of energy values corresponding to each frequency. Since the noise components $n_{l,m}$ are independent for different $m \in \{1, \dots, M\}$, components of \mathbf{R} are mutually independent. Hence, the conditional pdf of \mathbf{R} is

$$\begin{aligned} f_{\mathbf{R}|\mathbf{h}, s_k}(\mathbf{R}) &= \\ & \begin{cases} \left(\frac{R_k}{\xi}\right)^{\frac{L-1}{2}} e^{-(R_k + \xi)} I_{L-1}(2\sqrt{R_k \xi}) \prod_{n=1, n \neq k}^M \frac{R_n^{L-1} e^{-R_n}}{\Gamma(L)} & k \neq 0 \\ \frac{1}{[\Gamma(L)]^M} \prod_{n=1}^M R_n^{L-1} e^{-R_n} & k = 0 \end{cases} \end{aligned} \quad (7)$$

Then, the MAP rule that detects s_k for $k \neq 0$ is

$$\begin{cases} f_{\mathbf{R}|\mathbf{h}, s_k} > f_{\mathbf{R}|\mathbf{h}, s_m} & \forall m \neq 0, k \\ f_{\mathbf{R}|\mathbf{h}, s_k} > \frac{M(1-v)}{v} f_{\mathbf{R}|\mathbf{h}, s_0} \end{cases} \quad (8)$$

where we have used the fact that the prior probabilities of the transmitted signals are $p(s_m) = \frac{v}{M}$ for $m \neq 0$, and $p(s_0) = (1-v)$. Substituting (7) into to (8), the decision rule is simplified to:

$$\begin{cases} g_1(R_k) > g_1(R_m) & \forall m \neq k \\ g_1(R_k) > \frac{M(1-v)e^{\xi} \xi^{\frac{L-1}{2}}}{v(L-1)!} \end{cases} \quad (9)$$

where

$$g_1(R_k) = R_k^{-\frac{L-1}{2}} I_{L-1}(2\sqrt{R_k \xi}), \quad \xi > 0. \quad (10)$$

The following Lemma enables us to further simplify the detection rule.

Lemma 1: The function

$$g_1(x) = x^{-\frac{L-1}{2}} I_{L-1}(2\sqrt{x\xi}) \quad \text{for } x > 0, \xi > 0 \quad (11)$$

is a monotonically increasing function of x .

Proof: The derivative of the n^{th} order modified Bessel function is

$$\frac{dI_n(x)}{dx} = I_{n+1}(x) + \frac{n}{x} I_n(x). \quad (12)$$

Hence,

$$\begin{aligned} \frac{dI_{L-1}(2\sqrt{x\xi})}{dx} &= \sqrt{\frac{\xi}{x}} I_L(2\sqrt{x\xi}) + \frac{L-1}{2x} I_{L-1}(2\sqrt{x\xi}) \\ &> \frac{L-1}{2x} I_{L-1}(2\sqrt{x\xi}) \end{aligned} \quad (13)$$

where we use the fact that $\sqrt{\frac{\xi}{x}} I_L(2\sqrt{x\xi}) > 0$ for $x > 0$.

Then, the derivative of $g_1(\cdot)$ satisfies

$$\begin{aligned} \frac{dg_1(x)}{dx} &= -\frac{L-1}{2}x^{-\frac{L+1}{2}}I_{L-1}(2\sqrt{x\xi}) + x^{-\frac{L-1}{2}}\frac{dI_{L-1}(2\sqrt{x\xi})}{dx} \\ &> -\frac{L-1}{2}x^{-\frac{L+1}{2}}I_{L-1}(2\sqrt{x\xi}) + x^{-\frac{L-1}{2}}\frac{L-1}{2x}I_{L-1}(2\sqrt{x\xi}) \\ &= 0, \end{aligned}$$

proving that $g_1(x)$ is a monotonically increasing function of $x > 0$. ■

By the above result, the detection rule (9) further simplifies to

$$\begin{cases} R_k > R_m & \forall m \neq k \\ R_k > g_1^{-1}(T) \end{cases} \quad (14)$$

where $T = \frac{M(1-v)e^\xi \xi^{\frac{L-1}{2}}}{v(L-1)!}$. Since $g_1(\cdot)$ is a monotonically increasing function, the inverse $g_1^{-1}(\cdot)$ is well-defined. Note that (14) is the rule that detects the signal $s_k(t)$ for $k \neq 0$. The zero signal $s_0(t)$ is detected if

$$R_k < g_1^{-1}(T) \quad \forall k. \quad (15)$$

B. Probability of Error

In this section, we analyze the error probability of OOFSK modulation when MAP detection is used at the receiver. Suppose without loss of generality that $s_1(t)$ is the transmitted signal. Let $\tau = g_1^{-1}(T)$. Then the correct detection probability is

$$\begin{aligned} P_{c,1} &= P(R_1 > R_2, R_1 > R_3, \dots, R_1 > R_M, R_1 > \tau | s_1) \\ &= \int_\tau^\infty \left(\int_0^x f_{R_2|\mathbf{h},s_1}(t) dt \right)^{M-1} f_{R_1|\mathbf{h},s_1}(x) dx \\ &= \int_\tau^\infty \left(\int_0^x \frac{t^{L-1}}{\Gamma(L)} e^{-t} dt \right)^{M-1} f_{R_1|\mathbf{h},s_1}(x) dx. \end{aligned}$$

From [15], we have

$$\int_0^x \frac{1}{\Gamma(L)} t^{L-1} e^{-t} dt = 1 - e^{-x} \sum_{l=0}^{L-1} \frac{x^l}{l!} \quad (16)$$

Therefore, the correct detection probability can now be expressed as

$$P_{c,1} = \int_\tau^\infty \left[1 - e^{-x} \sum_{l=0}^{L-1} \frac{x^l}{l!} \right]^{M-1} f_{R_1|\mathbf{h},s_1}(x) dx$$

Using the binomial theorem, $P_{c,1}$ becomes

$$P_{c,1} = \int_\tau^\infty \sum_{n=0}^{M-1} (-1)^n \binom{M-1}{n} \left[\sum_{l=0}^{L-1} \frac{x^l}{l!} e^{-x} \right]^n f_{R_1|\mathbf{h},s_1}(x) dx$$

Using the multinomial theorem, we have the following expansion

$$\left[\sum_{l=0}^{L-1} \frac{x^l}{l!} e^{-x} \right]^n = e^{-nx} \sum_{i=0}^{n(L-1)} c_{in} x^i \quad (17)$$

where c_{in} is the coefficient of x^i in the expansion. c_{in} can be evaluated from the recursive equation [18]

$$c_{in} = \sum_{q=i-L+1}^i \frac{c_{q(n-1)}}{(i-q)!} I_{[0,(n-1)(L-1)]}(q) \quad (18)$$

where

$$I_{[a,b]}(q) = \begin{cases} 1, & a \leq q \leq b \\ 0, & \text{otherwise} \end{cases} \quad (19)$$

Using the multinomial expansion, $P_{c,1}$ becomes

$$\begin{aligned} P_{c,1} &= \sum_{n=0}^{M-1} (-1)^n \binom{M-1}{n} \sum_{i=0}^{n(L-1)} c_{in} \int_\tau^\infty x^i \left(\frac{x}{\xi} \right)^{\frac{L-1}{2}} \\ &\quad \times e^{-[(n+1)x+\xi]} I_{L-1}(2\sqrt{x\xi}) dx \end{aligned} \quad (20)$$

Let $\xi = a^2$ and $x = t^2$, then, $P_{c,1}$ can be written as

$$\begin{aligned} P_{c,1} &= \sum_{n=0}^{M-1} (-1)^n \binom{M-1}{n} \sum_{i=0}^{n(L-1)} c_{in} \\ &\quad \times \int_{\sqrt{\tau}}^\infty t^{2i} e^{-nt^2} \left(\frac{t}{a} \right)^{L-1} e^{-(t^2+a^2)} I_{L-1}(2at) 2t dt \\ &= \sum_{n=0}^{M-1} (-1)^n \binom{M-1}{n} \sum_{i=0}^{n(L-1)} 2c_{in} e^{a^2} a^{-(L-1)} \\ &\quad \times \left[\frac{a^{L-1} \Gamma(i+L)}{2(n+1)^{i+L} \Gamma(L)} e^{\frac{a^2}{n+1}} F\left(-i, L; \frac{-a^2}{n+1}\right) \right. \\ &\quad \left. - \int_0^{\sqrt{\tau}} t^{2i+L} e^{-(n+1)t^2} I_{L-1}(2at) dt \right] \end{aligned} \quad (21)$$

where $F(a, c; x)$ is the confluent hypergeometric function [16, Chap 10]. The probability of correct detection when signal $s_0(t)$ is transmitted is:

$$\begin{aligned} P_{c,0} &= P(R_1 < \tau, \dots, R_M < \tau | \mathbf{h}, s_0) \\ &= \left(1 - e^{-\tau} \sum_{l=0}^{L-1} \frac{\tau^l}{l!} \right)^M \end{aligned} \quad (22)$$

Hence, the probability of error as a function of the instantaneous signal-to-noise ratio is

$$P_e = 1 - (vP_{c,1} + (1-v)P_{c,0}). \quad (23)$$

Since the channel is assumed to be known, error probability in (23) is a function of the fading coefficients through $\chi = \sum_{l=1}^L |h_l|^2$. Hence, the average probability of error is obtained by computing

$$\bar{P}_e = \int_0^\infty P_e f_\chi(\chi) d\chi. \quad (24)$$

If h_l is a complex Gaussian random variable with mean value d_l and variance σ^2 and $\{h_l\}$ are mutually independent, χ is a chi-square random variable with $2L$ degrees of freedom and has a pdf given by

$$f_\chi(\chi) = \frac{1}{\sigma^2} \left(\frac{\chi}{\sigma^2} \right)^{\frac{L-1}{2}} e^{-\frac{\chi+\sigma^2}{\sigma^2}} I_{L-1} \left(\frac{2\sqrt{\chi\sigma^2}}{\sigma^2} \right) \quad (25)$$

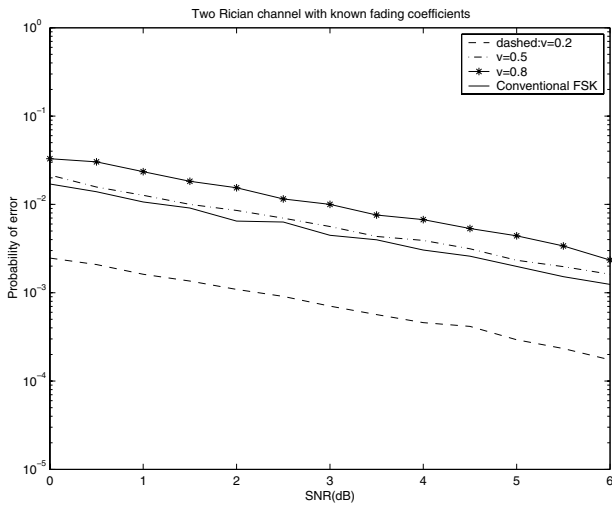


Fig. 1. Error probability vs. SNR for 4-OFSK signaling over two independent coherent Rician fading channels with equal Rician factor $K = \frac{1}{8}$. Duty factor values are $v = 1, 0.8, 0.5$, and 0.2 .

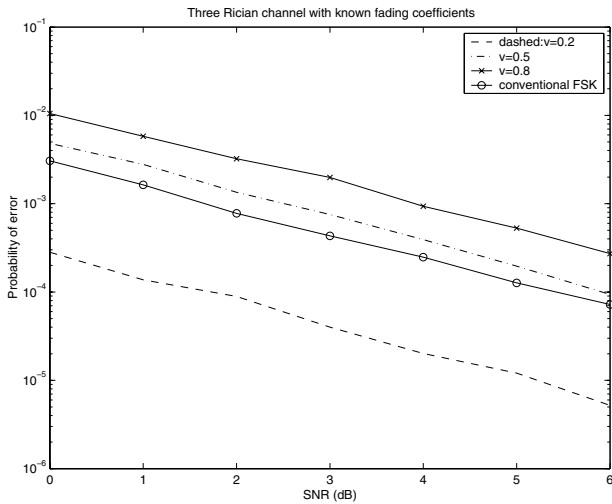


Fig. 2. Error probability vs. SNR for 4-OFSK signaling over three independent coherent Rician fading channels with equal Rician factor $K = \frac{1}{8}$. Duty factor values are $v = 1, 0.8, 0.5$, and 0.2 .

where, $s^2 = \sum_{l=1}^L |d_l|^2$.

When the fading coefficients $\{h_l\}$ are correlated, the average error probability \bar{P}_e can be obtained by evaluating the expected value of P_e with respect to the joint distribution of $(|h_1|, \dots, |h_L|)$, which involves L -fold integration. However, if $\{|h_l|\}$ are Nakagami- m distributed, closed-form expressions for $f_\chi(\chi)$ are provided in [19], which lead to a single integration.

Next, we present the simulation results. We define the Rician factor as $K = \frac{|d|^2}{\sigma^2}$ and correlation coefficient as $\rho = \frac{\text{cov}(h_i, h_j)}{\sqrt{\text{var}(h_i)\text{var}(h_j)}}$. Figures 1, 2, and 3 plot the probability of error curves as a function of SNR for 4-OFSK signaling over Rician fading channels with different number of receiver antennas and different duty factors. Two independent channels are considered in Fig. 1. Note that conventional FSK corre-

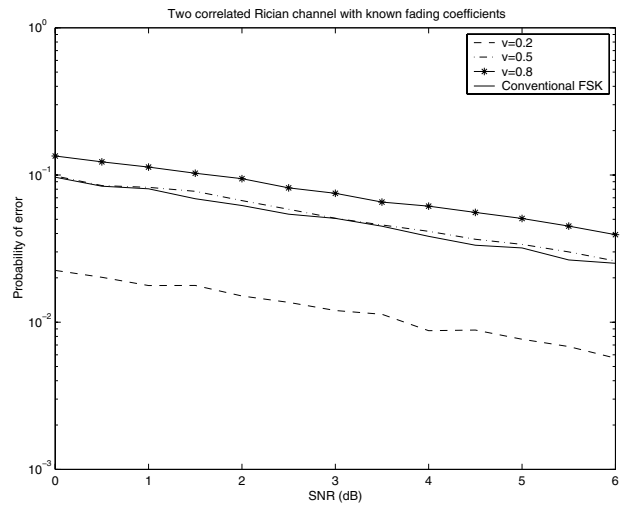


Fig. 3. Error probability vs. SNR for 4-OFSK signaling over two correlated coherent Rician fading channels with equal Rician factor $K = \frac{1}{8}$ and correlation coefficient $\rho = \frac{1}{4}$. Duty factor values are $v = 1, 0.8, 0.5$, and 0.2 .

sponds to OFSK with duty factor $v = 1$. In Fig. 2, three independent channels are assumed. In both figures, we observe an improvement in the error probability curves if the duty factor v of OFSK signaling is less than 0.5. When $v = 0.2$, we see approximately an order of magnitude improvement in the error performance. This results in substantial energy gains for fixed value of error probability, rendering OFSK signaling a very energy efficient transmission technique. It can also immediately be noted that due to increased diversity, having more antennas decreases the error rates. In Fig. 3, the case of two correlated channels is investigated. Although the performance is deteriorated due to correlation, OFSK with sufficiently small duty factor still considerably improves the error performance. It should be noted that having small duty factor means that FSK signals have high peak power but they are transmitted less frequently to satisfy the average power constraint. Consequently, having high peak power signals decrease the error rates.

IV. OFSK OVER NONCOHERENT FADING CHANNELS

A. Detection Rule

In the noncoherent channel case, we assume that the realizations of the fading coefficients $\{h_l\}$ are unknown at both the receiver and transmitter. The receiver is only equipped with the knowledge of the statistics of $\{h_l\}$. We further assume that $\{h_l\}$ are i.i.d. complex Gaussian random variables with $E\{h_l\} = d_l$ and $\text{var}\{h_l\} = \sigma^2$. Therefore, conditioned on $s_k(t)$ being the transmitted signal, $Y_{l,m}$ is a complex Gaussian random variable with

$$E\{Y_{l,m}|s_k\} = \begin{cases} Ad_l e^{j\theta_k} & m = k \\ 0 & m \neq k \end{cases},$$

$$\text{var}\{Y_{l,m}|s_k\} = \begin{cases} A^2 \sigma^2 + 1 & m = k \\ 1 & m \neq k \end{cases}.$$

Similarly as in the coherent case, we combine the energies of the m^{th} frequency components across the antennas, and obtain $R_m = \sum_{l=1}^L R_{m,l}$. Conditioned on transmitted signal $s_k(t)$, R_m is a chi-square random variable with the following pdf:

$$f_{R_m|s_k}(R_m) = \begin{cases} \frac{1}{\sigma_y^2} \left(\frac{R_m}{\xi}\right)^{\frac{L-1}{2}} e^{-\frac{R_m+\xi}{\sigma_y^2}} I_{L-1}\left(\frac{2\sqrt{R_m\xi}}{\sigma_y^2}\right) & m = k \\ \frac{R_m^{L-1}}{\Gamma(L)} e^{-R_m} & m \neq k \end{cases} \quad (26)$$

where $\xi = A^2 \sum_{l=1}^L |d_l|^2$ and $\sigma_y^2 = A^2\sigma^2 + 1$. The vector $\mathbf{R} = [R_1, \dots, R_M]$ has the following conditional joint pdf

$$f_{\mathbf{R}|s_k}(\mathbf{R}) = \begin{cases} \frac{1}{\sigma_y^2} \left(\frac{R_k}{\xi}\right)^{\frac{L-1}{2}} e^{-\frac{R_k+\xi}{\sigma_y^2}} I_{L-1}\left(\frac{2\sqrt{R_k\xi}}{\sigma_y^2}\right) \prod_{n=1, n \neq k}^M \frac{R_n^{L-1} e^{-R_n}}{\Gamma(L)} & k \neq 0 \\ \frac{1}{[\Gamma(L)]^M} \prod_{n=1}^M R_n^{L-1} e^{-R_n} & k = 0 \end{cases}$$

The MAP decision rule that detects s_k for $k \neq 0$ is

$$\begin{cases} f_{\mathbf{R}|s_k} > f_{\mathbf{R}|s_m} & \forall m \neq 0, k \\ f_{\mathbf{R}|s_k} > \frac{M(1-v)}{v} f_{\mathbf{R}|s_0} \end{cases} \quad (27)$$

Similarly as in Section III, it can be easily shown that

$$g_2(R_k) = R_k^{-\frac{L-1}{2}} e^{\frac{R_k A^2 \sigma^2}{\sigma_y^2}} I_{L-1}\left(\frac{2\sqrt{R_k\xi}}{\sigma_y^2}\right), \quad \xi > 0 \quad (28)$$

is a monotonically increasing function. With this observation, the decision rule in (27) simplifies to

$$\begin{cases} R_k > R_m & \forall m \neq k \\ R_k > g_2^{-1}(T_2) \end{cases} \quad (29)$$

where $T_2 = \frac{M(1-v)\sigma_y^2 \xi^{\frac{L-1}{2}} e^{\frac{\xi}{\sigma_y^2}}}{v\Gamma(L)}$. Note that s_0 is the detected signal if $R_k < g_2^{-1}(T_2)$ for all k .

B. Probability of Error

We first assume that $s_1(t)$ is transmitted. Let $\tau_2 = g_2^{-1}(T_2)$. Then, the probability of correct detection is

$$P_{c,1} = P(R_2 > R_1, R_3 > R_1, \dots, R_M > R_1, R_1 > \tau_2 | s_1) \quad (30)$$

Following an approach similar to that in Section III, we have

$$\begin{aligned} P_{c,1} &= \sum_{n=0}^{M-1} (-1)^n \binom{M-1}{n} \sum_{i=0}^{n(L-1)} c_{in} \int_{\tau_2}^{\infty} x^i e^{-nx} f_{R_1|s_1}(x) dx \\ &= \sum_{n=0}^{M-1} (-1)^n \binom{M-1}{n} \sum_{i=0}^{n(L-1)} c_{in} \int_{\tau_2}^{\infty} x^i e^{-nx} \\ &\quad \times \frac{1}{\sigma_y^2} \left(\frac{x}{\xi}\right)^{\frac{L-1}{2}} e^{-\frac{x+\xi}{\sigma_y^2}} I_{L-1}\left(\frac{2\sqrt{x\xi}}{\sigma_y^2}\right) dx \\ &= \sum_{n=0}^{M-1} (-1)^n \binom{M-1}{n} \sum_{i=0}^{n(L-1)} c_{in} \frac{\xi^{-\frac{L-1}{2}} e^{-\frac{\xi}{\sigma_y^2}}}{\sigma_y^2} \\ &\quad \times \left[\frac{\xi^{\frac{L-1}{2}} (i+L)!}{2(1+n\sigma_y^2)^{\frac{i+L}{2}} \sigma_y^{L-2-i} L!} F\left(-i, L, \frac{\xi}{\sigma_y^2(1+n\sigma_y^2)}\right) \right. \\ &\quad \left. \times e^{\frac{\xi}{\sigma_y^2(1+n\sigma_y^2)}} - \int_0^{\tau_2} x^{\frac{2i+L-1}{2}} e^{-\frac{1+n\sigma_y^2}{\sigma_y^2}x} I_{L-1}\left(\frac{2\sqrt{x\xi}}{\sigma_y^2}\right) dx \right] \end{aligned}$$

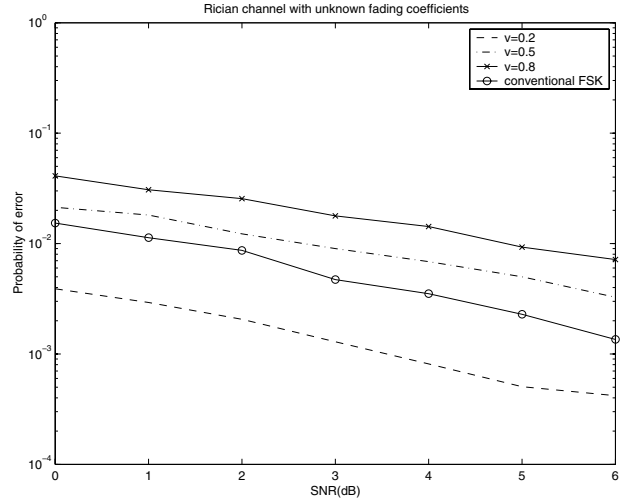


Fig. 4. Error probability vs. SNR for 4-OFSK signaling over two independent noncoherent Rician fading channels with equal Rician factor $K = \frac{1}{8}$.

If $s_0(t)$ is the transmitted signal, the probability of correct detection is

$$\begin{aligned} P_{c,0} &= P(R_1 < \tau_2, \dots, R_M < \tau_2 | s_0) \\ &= \left(1 - e^{-\tau} \sum_{l=0}^{L-1} \frac{\tau^l}{l!}\right)^M \end{aligned} \quad (31)$$

Finally, the average probability of error is

$$P_e = 1 - (vP_{c,1} + (1-v)P_{c,0}). \quad (32)$$

Figures 4, 5, and 6 provide the simulation results of error probability when 4-OFSK signals are transmitted over noncoherent Rician fading channels. In Figs. 4 and 5, the channels are assumed to be independent. In these figures, it is seen that OFFSK signaling with $v = 0.8$ and $v = 0.5$ have worse error performance when compared to that of conventional FSK (OFSK with $v = 1$). As evidenced in the graph of $v = 0.2$, if the duty factor is sufficiently decreased, and hence consequently the peak power is increased, we start seeing improvements. Since fading is not known in the noncoherent case, the advantage of using OFFSK signaling is twofold. Having low duty cycle allows the FSK signals to have high peak power which is especially beneficial when channel characteristics are unknown. In addition, when the zero signal $s_0(t)$ is sent, the received signal is composed of additive noise and is free of fading coefficients. Finally, Fig. 6 plots the error probabilities when 4-OFSK signals are sent over two correlated noncoherent channels. These curves are obtained when the detection rule (29) derived for independent channels are employed at the receiver. We note that the performance degrades due to correlation in noncoherent channels as well. Similar conclusions about OFFSK modulation are drawn.

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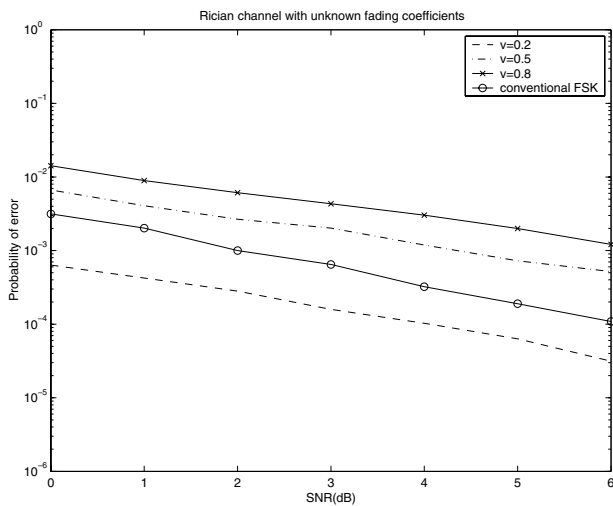


Fig. 5. Error probability vs. SNR for 4-OFSK signaling over three independent noncoherent Rician fading channels with equal Rician factor $K = \frac{1}{8}$.

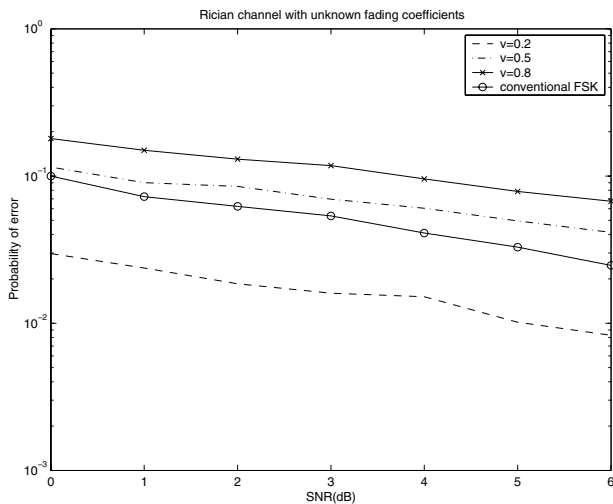


Fig. 6. Error probability vs. SNR for 4-OFSK signaling over two correlated noncoherent Rician fading channels with equal Rician factor $K = \frac{1}{8}$.

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