

# Error Probability Analysis of Peaky Signaling over Fading Channels

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**Abstract**—In this paper, the performance of signaling strategies with high peak-to-average power ratio is analyzed in both coherent and noncoherent fading channels. Two recently proposed modulation schemes, namely on-off binary phase-shift keying and on-off quaternary phase-shift keying, are considered. For these modulation formats, the optimal decision regions used at the detector are identified and analytical expressions for the error probabilities are obtained. Numerical techniques are employed to compute the error probabilities. It is concluded that increasing the peakedness of the signals results in reduced error rates for a given power level and hence improve the energy efficiency.

## I. INTRODUCTION

In wireless communications, when the receiver and transmitter have only imperfect knowledge of the channel conditions, efficient transmission strategies have a markedly different structure than those employed over perfectly known channels. For instance, Abou-Faycal *et al.* [1] studied the noncoherent Rayleigh fading channel where the receiver and transmitter has no channel side information, and showed that the capacity-achieving input amplitude distribution is discrete with a finite number of mass points. It has also been shown that there always exists a mass point at the origin. These results indicate that unlike perfectly known channels where a Gaussian input is optimal and a continuum of amplitude levels are available for transmission, only finitely many amplitude levels with one level at the origin should be used for transmission over noncoherent Rayleigh channels. The discreteness of the optimal amplitude distribution has also been shown over other noncoherent channels (see e.g., [2], [3], [4], [6], and references therein).

Another key result for noncoherent channels is the requirement of transmission with high peak-to-average power ratio in the low signal-to-noise ratio (SNR) regime. In noncoherent Rayleigh channels [1], the optimal amplitude has two mass points for low SNR values, and the nonzero mass migrates away from the origin, increasing the peak power, as SNR decreases. Indeed, this behavior has been shown in a more general setting in [5] where *flash signaling* is proven to be the necessary form of transmission to achieve the capacity in the low-SNR regime.

The impact upon the channel capacity of using signals with limited peakedness is investigated in [7], [8], [9], and [10]. In [7], two types of signaling schemes are defined: on-off binary-shift keying (OOBPSK) and on-off quaternary phase-shift keying (OOQPSK). These modulations are obtained by

overlaying on-off keying on phase-shift keying. The peakedness of these signals are controlled by changing the probability of no transmission. OOQPSK is shown to be an optimally efficient modulation scheme for transmission over noncoherent Rician fading channels in the low-SNR regime.

Motivated by the above-mentioned results, we study in this paper the error performance of using signals with high peak-to-average power ratios (PAR) over fading channels. Potential gains in error performance are anticipated in both coherent and noncoherent fading channels as increasing the PAR without changing the average power consumption increases the minimum distance between the constellation points. This is achieved by increasing the probability of no signal transmission, which potentially decreases the data rate. One possible application in which such transmission techniques are desirable is wireless sensor networks where low-data rate transmissions with low energy consumption are required. For instance, the specifications of a wireless microsensor system given in [11] indicate that the microsensors need to operate for 5-10 years with one AA size battery.

The rest of the paper is organized as follows. We describe the channel model and modulation techniques in Section II. We study the error probability in coherent fading channels in Section III while the performance in noncoherent fading channels is analyzed in Section IV. Finally, Section V includes our conclusions.

## II. SYSTEM MODEL

We consider the following single-input, single-output fading channel

$$y_i = h_i x_i + n_i \quad i = 1, 2, \dots, \quad (1)$$

where  $x_i$  is the complex channel input,  $y_i$  is the complex channel output,  $h_i$  is the channel fading coefficient, and  $n_i$  is the additive Gaussian noise. Note that the transmitted signal experiences flat fading and hence the it is assumed that the delay spread of the channel is much smaller than the symbol duration. The fading process  $\{h_i\}$  is assumed to be a stationary, ergodic, and proper complex random process. Moreover,  $\{n_i\}$  is a sequence of independent and identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian random variables with variance  $E\{|n_i|^2\} = N_0$ . It is further assumed that the channel input, fading coefficients,

and additive noise samples are mutually independent of each other.

We consider that the transmitter employs OOBPSK or OOQPSK modulation for transmission. An OOBPSK signal, parametrized by  $0 < \nu \leq 1$ , has the following constellation points together with their transmission probabilities:

$$x_k = \begin{cases} 0 & \text{with prob. } (1 - \nu), \quad k = 0 \\ +\sqrt{\frac{P}{\nu}} & \text{with prob. } (\nu/2), \quad k = 1 \\ -\sqrt{\frac{P}{\nu}} & \text{with prob. } (\nu/2), \quad k = 2 \end{cases} \quad (2)$$

Similarly, an OOQPSK signal has the following constellation points with the corresponding transmission probabilities:

$$x_k = \begin{cases} 0 & \text{with prob. } (1 - \nu), \quad k = 0 \\ \sqrt{\frac{P}{2\nu}}(\pm 1 \pm j) & \text{with prob. } (\nu/4), \quad k = 1, 2, 3, 4 \end{cases} \quad (3)$$

If the above modulations are adopted, the transmitter either sends no signal with probability  $1 - \nu$  or one of the BPSK or QPSK constellation points with probability  $\nu$ . Hence,  $\nu$  is the duty cycle of the transmission. Note that these definitions can straightforwardly be extended to use phase-shift keying signals with larger constellation sizes. Note also that both signals have an average power of  $P$  and a peak power of  $\frac{P}{\nu}$ , and hence a peak-to-average power ratio (PAR) of  $\frac{1}{\nu}$ . Limitations on the PAR of the signaling scheme may be dictated by regulations or system component specifications.

### III. ERROR PROBABILITY OVER COHERENT FADING CHANNELS

In this section, we study the error probability of OOBPSK and OOQPSK signaling schemes. Here, it is assumed that the receiver perfectly knows the instantaneous realizations of the fading coefficients  $\{h_k\}$  whereas the transmitter has no such knowledge. Our focus will be on uncoded systems so that error performance improvements obtained by using peaky signals can be identified in the absence of coding gain achievements.

For the detection of the signals, maximum a posteriori probability (MAP) decision rule, which minimizes the probability of error, is employed at the receiver. It is assumed that symbol-by-symbol detection is performed. In MAP detection, signal point  $x_k$  is chosen as the detected signal if

$$p_k f_{y|x_k, h}(y|x_k, h) > p_l f_{y|x_l, h}(y|x_l, h) \quad \forall l \neq k \quad (4)$$

where  $p_k$  and  $p_l$  denote the prior transmission probabilities of the signal points  $x_k$  and  $x_l$  respectively, and

$$f_{y|x_k, h}(y|x_k, h) = \frac{1}{\pi N_0} \exp\left(-\frac{|y - hx_k|^2}{N_0}\right) \quad (5)$$

is the conditional probability density function of the output given  $x_k$  and  $h$ . Using the property that phase-shift keying signals have the same energy, the detection rule can be simplified as follows. The signal  $x_k$  for  $k \neq 0$  is the detected signal if

$$\Re(yx_k^*) > \Re(yx_l^*) \quad \forall l \neq k, 0 \quad \text{and} \quad \Re(yx_k^*) > T \quad (6)$$

where

$$T = \frac{1}{2} \frac{|h|P}{\nu} + \frac{N_0}{2|h|} \ln\left(\frac{\xi(1-\nu)}{\nu}\right) \quad (7)$$

is a threshold value with  $\xi = 2$  for OOBPSK signaling and  $\xi = 4$  for OOQPSK signaling. In the above formulation,  $\Re(z)$  denotes the real part of the complex scalar  $z$ , and  $z^*$  is the complex conjugate of  $z$ . The signal point at the origin,  $x_0$ , is the detected signal if

$$\Re(yx_l^*) < T \quad \forall l \neq 0. \quad (8)$$

Since OOBPSK and OOQPSK signaling schemes have circular symmetry, the phase  $\theta_h$  of fading coefficients has no effect on the error probability as long as the receiver, knowing perfectly the phase rotations introduced by the channel, removes any phase offset and uses  $\tilde{y} = ye^{-\theta_h}$  at the detector. Hence, the detection rules (6) and (8) are obtained by assuming without loss of generality that  $\theta_h = 0$  and hence  $h = |h|$ .

#### A. OOBPSK Signaling

When OOBPSK signals are sent, the imaginary component of  $y = y_r + jy_i$  does not carry any information and the decision is based on the real part  $y_r$ . Decision regions in the two-dimensional received signal space are given as follows:

$$\mathbf{R}_0 = \{y : |y_r| < T_b\} \quad (9)$$

$$\mathbf{R}_1 = \{y : y_r > 0 \text{ and } y_r > T_b\} \quad (10)$$

$$\mathbf{R}_2 = \{y : y_r < 0 \text{ and } y_r < -T_b\} \quad (11)$$

where

$$T_b = T \sqrt{\frac{\nu}{P}} = \frac{1}{2} \sqrt{\frac{|h|^2 P}{\nu}} + \frac{1}{2} \sqrt{\frac{N_0^2 \nu}{|h|^2 P}} \ln\left(\frac{2(1-\nu)}{\nu}\right)$$

and  $\mathbf{R}_k$  denotes the decision region for signal point  $x_k$ . After the decision regions are identified, the probability of symbol error expressions are easily obtained. The error probability when a nonzero signal is transmitted is given by

$$P_{e|x_1, |h|} = P_{e|x_2, |h|} = Q\left(\sqrt{\frac{2|h|^2 P}{\nu N_0}} - T_b \sqrt{\frac{2}{N_0}}\right) \quad (12)$$

where  $Q(\cdot)$  is the Gaussian Q-function. The error probability when  $x_0$  is sent is

$$P_{e|x_0, |h|} = 2Q\left(T_b \sqrt{\frac{2}{N_0}}\right). \quad (13)$$

Therefore, the error probability as a function of the instantaneous realization of the fading coefficient is given by

$$P_{e|h} = (1-\nu)2Q\left(T_b \sqrt{\frac{2}{N_0}}\right) + \nu Q\left(\sqrt{\frac{2|h|^2 P}{\nu N_0}} - T_b \sqrt{\frac{2}{N_0}}\right). \quad (14)$$

It is important to note that the above analysis assumes  $T_b > 0$ . However, if  $\nu > 2/3$ , we have  $T_b \leq 0$  for sufficiently small values of  $P$ . In this case, it can be easily seen that

$$P_{e|h} = (1-\nu) + \nu Q\left(\sqrt{\frac{2|h|^2 P}{\nu N_0}}\right). \quad (15)$$

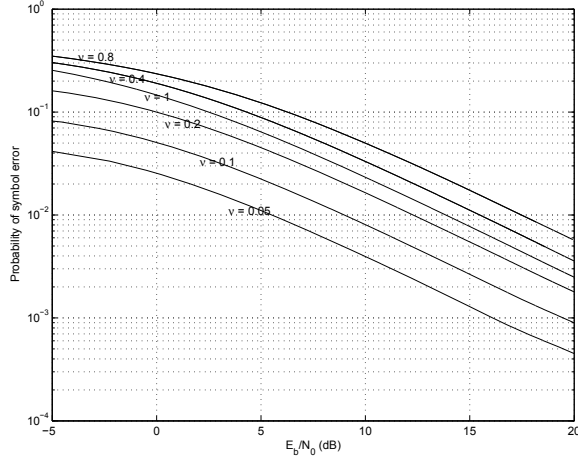


Fig. 1. Probability of symbol error vs.  $E_b/N_0$  for OOBPSK signaling with different duty factor values,  $\nu$ , in coherent Rayleigh fading channels with  $E\{|h|^2\} = 1$ .

Finally, the average probability of error is obtained from

$$P_e = \int_0^\infty P_{e| |h|} dF_{|h|}(|h|) \quad (16)$$

where  $F_{|h|}$  is the distribution function of the fading magnitude. Figure 1 plots the error probability curves for OOBPSK signaling in the Rayleigh fading channel with  $E\{|h|^2\} = 1$ . The integral in (16) is computed using the Gauss-Laguerre quadrature technique. In ordinary BPSK modulation, each symbol carries one bit. It is important to note that in OOBPSK modulation, the maximum number of bits that can be carried by the symbols is equal to the entropy

$$H(\nu) = \nu \log_2(2/\nu) + (1 - \nu) \log_2(1/(1 - \nu)) \quad (17)$$

which decreases to zero as  $\nu \rightarrow 0$ . Hence, for fair comparison, Fig. 1 plots the curves as a function of the SNR normalized by the entropy of the OOBPSK source, giving the SNR per bit. It is observed from the figure that if the peakedness of the input signals is increased sufficiently (e.g.,  $\nu = 0.2, 0.1, 0.05$ ), significant improvements in error performance are achieved over ordinary BPSK (i.e., OOBPSK with  $\nu = 1$ ) performance.

### B. OOQPSK Signaling

When OOQPSK signals are employed for transmission, the decision regions are

$$\mathbf{R}_0 = \{y : |y_r + y_i| < T_q \text{ and } |y_r - y_i| < T_q\} \quad (18)$$

$$\mathbf{R}_1 = \{y : y_r > 0, y_i > 0, \text{ and } y_r + y_i > T_q\} \quad (19)$$

$$\mathbf{R}_2 = \{y : y_r < 0, y_i > 0, \text{ and } -y_r + y_i > T_q\} \quad (20)$$

$$\mathbf{R}_3 = \{y : y_r < 0, y_i < 0, \text{ and } -y_r - y_i > T_q\} \quad (21)$$

$$\mathbf{R}_4 = \{y : y_r > 0, y_i < 0, \text{ and } y_r - y_i > T_q\} \quad (22)$$

where

$$T_q = T \sqrt{\frac{2\nu}{P}} = \sqrt{\frac{|h|^2 P}{2\nu}} + \sqrt{\frac{N_0^2 \nu}{2|h|^2 P}} \ln\left(\frac{4(1-\nu)}{\nu}\right). \quad (23)$$

In this case, it is more convenient to initially obtain the correct detection probabilities. We first assume that  $T_q > 0$ . When  $x_1$  is the transmitted signal point, the correct detection probability is

$$P_{c|x_1, |h|} = P(y \in R_1) \quad (24)$$

$$= P(y_r > 0, y_i > 0, y_r + y_i > T_q) \quad (25)$$

$$= P(y_r > T_q)P(y_i > 0) + P(0 < y_r < T_q, y_r + y_i > T_q)$$

$$= \left(1 - Q\left(\sqrt{\frac{|h|^2 P}{\nu N_0}}\right)\right) Q\left(\frac{T_q - \sqrt{\frac{|h|^2 P}{2\nu}}}{\sqrt{N_0/2}}\right)$$

$$+ \int_0^{T_q} Q\left(\frac{T_q - x - \sqrt{\frac{|h|^2 P}{2\nu}}}{\sqrt{N_0/2}}\right) \frac{e^{-\frac{(x - \sqrt{\frac{|h|^2 P}{2\nu}})^2}{N_0}}}{\sqrt{\pi N_0}} dx. \quad (26)$$

Similarly as in the previous section, it can be seen that if  $\nu > 0.8$ , for sufficiently small  $P$  we have  $T_q \leq 0$  for which case

$$P_{c|x_1, |h|} = P(y_r > 0, y_i > 0) \quad (27)$$

$$= \left(1 - Q\left(\sqrt{\frac{|h|^2 P}{\nu N_0}}\right)\right)^2. \quad (28)$$

Due to the circular symmetry of the signaling scheme, the same correct detection probabilities as in (26) and (28) are obtained when signal points  $x_2, x_3$ , and  $x_4$  are transmitted. If  $x_0$  is sent, the correct detection probability is

$$P_{c|x_0, |h|} = \begin{cases} 4 \int_0^{T_q} \left(\frac{1}{2} - Q\left(\frac{T_q - x}{\sqrt{N_0/2}}\right)\right) \frac{e^{-\frac{x^2}{N_0}}}{\sqrt{\pi N_0}} dx & \text{if } T_q > 0 \\ 0 & \text{if } T_q \leq 0 \end{cases}$$

Now, the error probability as a function of the fading coefficients and the average error probability are given by

$$P_{e| |h|} = 1 - ((1 - \nu)P_{c|x_0, |h|} + \nu P_{c|x_1, |h|}), \quad (29)$$

and

$$P_e = \int_0^\infty P_{e| |h|} dF_{|h|}(|h|), \quad (30)$$

respectively, where  $F_{|h|}$  is the distribution function of the fading magnitude. Fig. 2 plots the error probability curves for OOQPSK signaling again as a function of the SNR per bit (i.e., SNR normalized by the entropy of OOQPSK signals) in the coherent Rayleigh fading channel with  $E\{|h|^2\} = 1$ . It is seen that error performance improves if  $\nu \leq 0.5$  and significant gains are obtained when  $\nu = 0.1$  which requires a 10-fold increase in the peak-to-average power ratio when compared to ordinary QPSK signaling. It is interesting to note that if  $\nu < 0.8$ , the threshold value  $T_q \rightarrow \infty$  as  $P \rightarrow 0$ . Therefore, for sufficiently small  $P$ , the zero signal,  $x_0$ , is always chosen as the detected signal and the error probability becomes  $\nu$ . Indeed, it is observed in Fig. 2 that the error curves corresponding to OOQPSK signaling with  $\nu < 0.8$  approach to  $\nu$  as SNR decreases. If  $\nu > 0.8$ ,  $T_q \rightarrow -\infty$  as  $P \rightarrow 0$ . When  $T_q \leq 0$ , the zero signal is never detected. Note that this behavior is also exhibited if OOBPSK modulation is used.

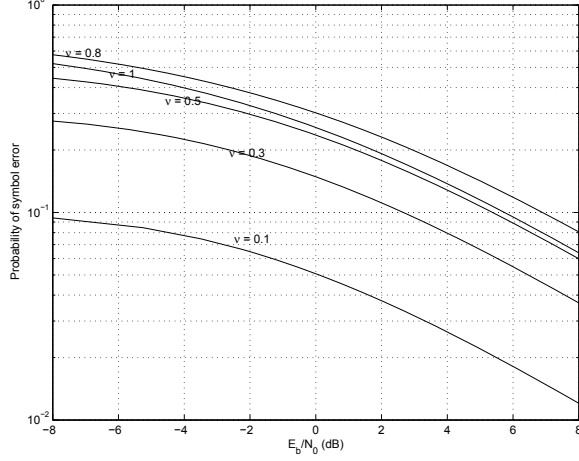


Fig. 2. Probability of symbol error vs.  $E_b/N_0$  for OOQPSK signaling with different duty factor values,  $\nu$ , in coherent Rayleigh fading channels with  $E\{|h|^2\} = 1$ .

#### IV. ERROR PROBABILITY OVER NONCOHERENT FADING CHANNELS

In this section, we consider the scenario in which neither the transmitter nor the receiver knows the instantaneous realizations of the fading coefficients. We consider a fast Rician fading environment and hence  $\{h_k\}$  is a sequence of i.i.d. proper complex Gaussian random variables with mean  $E\{h_k\} = m$  and variance  $E\{|h_k|^2\} = \gamma^2$ . It is assumed that channel statistics and hence the values of  $m$  and  $\gamma^2$  are assumed to be known at the transmitter and receiver. The conditional output probability density function given the input is

$$f_{y|x}(y|x) = \frac{1}{\pi(\gamma^2|x|^2 + N_0)} e^{-\frac{|y-mx|^2}{\gamma^2|x|^2 + N_0}}. \quad (31)$$

Similarly as in Section III, MAP detection is employed at the detector. Hence,  $x_k$  is the detected signal if

$$p_k f_{y|x_k}(y|x_k) > p_l f_{y|x_l}(y|x_l) \quad \forall l \neq k. \quad (32)$$

The decision rule (32) is simplified to yield the following rule:  $x_k$  for  $k \neq 0$  is the detected signal if

$$\Re(yx_k^*) > \Re(yx_l^*) \quad \forall l \neq k, 0 \quad (33)$$

and

$$\frac{\gamma^2 P}{\nu} |y|^2 + 2N_0|m| \Re(yx_k^*) > T \quad (34)$$

where

$$T = N_0 \frac{P}{\nu} |m|^2 + N_0 \left( \gamma^2 \frac{P}{\nu} + N_0 \right) \ln \left( \frac{\xi(1-\nu)}{\nu} \left( \gamma^2 \frac{P}{\nu N_0} + 1 \right) \right) \quad (35)$$

with  $\xi = 2$  for OOBPSK signaling and  $\xi = 4$  for OOQPSK signaling. The signal point  $x_0$  is the detected signal if

$$\frac{\gamma^2 P}{\nu} |y|^2 + 2N_0|m| \Re(yx_l^*) < T \quad \forall l \neq 0. \quad (36)$$

Using the same arguments as in Section III, we obtain the above decision rules by assuming without loss generality that  $m = |m|$ . Otherwise, in all the decision rules,  $y$  must be replaced by  $\tilde{y} = ye^{-\theta_m}$  where  $\theta_m$  is the phase of fading mean  $m$ . It is further assumed that  $|m| > 0$  as phase-shift keying cannot be employed to carry information over an unknown Rayleigh fading channel.

#### A. OOBPSK Signaling

We first analyze the performance of OOBPSK signaling. The decision regions for this signaling are

$$\mathbf{R}_0 = \{y : (y_r + A_b) + y_i^2 < C_b \text{ and } (y_r - A_b) + y_i^2 < C_b\},$$

$$\mathbf{R}_1 = \{y : y_r > 0 \text{ and } (y_r + A_b)^2 + y_i^2 > C_b\},$$

$$\mathbf{R}_2 = \{y : y_r < 0 \text{ and } (y_r - A_b)^2 + y_i^2 > C_b\}$$

where

$$A_b = \frac{N_0|m|}{\gamma^2} \sqrt{\frac{\nu}{P}} \quad \text{and} \quad C_b = T \frac{\nu}{\gamma^2 P} + A_b^2 \quad (37)$$

with  $T$  given in (35). It is interesting to note that unlike the coherent case in Section III-A, both the real and imaginary components of the received signal point  $y$  are needed to decide on the transmitted signal when the channel is not known.

Similarly as before, we first consider the case in which  $T > 0$ . When a nonzero signal is transmitted, the correct detection probability is

$$\begin{aligned} P_{c|x_1} &= P_{c|x_2} \quad (38) \\ &= P(y_r > \sqrt{C_b} - A_b) \\ &\quad + P(0 \leq y_r \leq \sqrt{C_b} - A_b, (y_r + A_b)^2 + y_i^2 > C_b) \\ &= Q \left( \frac{\sqrt{C_b} - A_b - \sqrt{\frac{|m|^2 P}{\nu}}}{\sqrt{\frac{\gamma^2 P}{2\nu} + \frac{N_0}{2}}} \right) \\ &\quad + 2 \int_0^{\sqrt{C_b} - A_b} Q \left( \frac{\sqrt{C_b - (x + A_b)^2}}{\sqrt{\frac{\gamma^2 P}{2\nu} + \frac{N_0}{2}}} \right) \frac{e^{-\frac{(x - \sqrt{\frac{|m|^2 P}{\nu}})^2}{\frac{\gamma^2 P}{\nu} + N_0}}}{\sqrt{\pi(\frac{\gamma^2 P}{\nu} + N_0)}} dx. \end{aligned} \quad (39)$$

If  $T < 0$ ,

$$P_{c|x_1} = P_{c|x_2} = 1 - Q \left( \frac{\sqrt{\frac{|m|^2 P}{\nu}}}{\sqrt{\frac{\gamma^2 P}{2\nu} + \frac{N_0}{2}}} \right). \quad (40)$$

When  $x_0$  is transmitted, the correct detection probability is given by

$$P_{c|x_0} = \begin{cases} 4 \int_0^{\sqrt{C_b} - A_b} \left( \frac{1}{2} - Q \left( \frac{\sqrt{C_b - (x + A_b)^2}}{\sqrt{\frac{\gamma^2 P}{2\nu} + \frac{N_0}{2}}} \right) \right) \frac{e^{-\frac{x^2}{N_0}}}{\sqrt{\pi N_0}} dx & \text{if } T > 0 \\ 0 & \text{if } T \leq 0 \end{cases}$$

Finally, the error probability is obtained from

$$P_e = 1 - ((1 - \nu)P_{c|x_0} + \nu P_{c|x_1}). \quad (41)$$

#### B. OOQPSK Signaling

The decision regions for OOQPSK signals are given below:

$$\mathbf{R}_0 = \{y : (y_r + (-1)^m A_q)^2 + (y_i + (-1)^n A_q)^2 < C_q, \forall m, n \in \{0, 1\}\}$$

$$\mathbf{R}_1 = \{y : y_r > 0, y_i > 0, \text{ and } (y_r + A_q)^2 + (y_i + A_q)^2 > C_q\}$$

$$\mathbf{R}_2 = \{y : y_r < 0, y_i > 0, \text{ and } (y_r - A_q)^2 + (y_i + A_q)^2 > C_q\}$$

$$\mathbf{R}_3 = \{y : y_r < 0, y_i < 0, \text{ and } (y_r - A_q)^2 + (y_i - A_q)^2 > C_q\}$$

$$\mathbf{R}_4 = \{y : y_r > 0, y_i < 0, \text{ and } (y_r + A_q)^2 + (y_i - A_q)^2 > C_q\}$$

where

$$A_q = \frac{N_0|m|}{\gamma^2} \sqrt{\frac{\nu}{2P}} \quad \text{and} \quad C_q = T \frac{\nu}{\gamma^2 P} + 2A_q^2 \quad (42)$$

with  $T$  given in (35). As before, we first express the correct detection probabilities. If a nonzero signal point is sent and  $T > 0$ , we have

$$\begin{aligned} P_{c|x_1} &= P(y_r > 0, y_i > 0, \text{ and } (y_r + A_q)^2 + (y_i + A_q)^2 > C_q) \\ &= P(y_r > D_q)P(y_i > 0) \\ &\quad + P(0 \leq y_r \leq D_q, y_i > 0, (y_r + A_q)^2 + (y_i + A_q)^2 > C_q) \\ &= \left(1 - Q\left(\frac{\sqrt{\frac{|m|^2 P}{2\nu}}}{\sqrt{\frac{\gamma^2 P}{2\nu} + \frac{N_0}{2}}}\right)\right) Q\left(\frac{D_q - \sqrt{\frac{|m|^2 P}{2\nu}}}{\sqrt{\frac{\gamma^2 P}{2\nu} + \frac{N_0}{2}}}\right) \\ &\quad + \int_0^{D_q} Q\left(\frac{\sqrt{C_q - (x + A_q)^2} - A_q - \sqrt{\frac{|m|^2 P}{2\nu}}}{\sqrt{\frac{\gamma^2 P}{2\nu} + \frac{N_0}{2}}}\right) \\ &\quad \times \frac{\left(x - \sqrt{\frac{|m|^2 P}{2\nu}}\right)^2}{\frac{\gamma^2 P}{\nu} + N_0} \\ &\quad \times \frac{e^{-\frac{x^2}{\frac{\gamma^2 P}{\nu} + N_0}}}{\sqrt{\pi\left(\frac{\gamma^2 P}{\nu} + N_0\right)}} dx \end{aligned} \quad (43)$$

where  $D_q = \sqrt{C_q - A_q^2} - A_q$ . If  $T < 0$ ,

$$P_{c|x_1} = \left(1 - Q\left(\frac{\sqrt{\frac{|m|^2 P}{2\nu}}}{\sqrt{\frac{\gamma^2 P}{2\nu} + \frac{N_0}{2}}}\right)\right)^2. \quad (44)$$

If  $x_0$  is the transmitted signal,

$$P_{c|x_0} = \begin{cases} 4 \int_0^{D_q} \left(\frac{1}{2} - Q\left(\frac{\sqrt{C_q - (x + A_q)^2} - A_q}{\sqrt{\frac{N_0}{2}}}\right)\right) \frac{e^{-\frac{x^2}{N_0}}}{\sqrt{\pi N_0}} dx & \text{if } T > 0 \\ 0 & \text{if } T \leq 0 \end{cases}$$

Finally, the error probability is obtained from

$$P_e = 1 - ((1 - \nu)P_{c|x_0} + \nu P_{c|x_1}). \quad (45)$$

Fig. 3 plots the error probability curves for OQPSK signaling as a function of SNR per bit in the noncoherent Rician fading channel with Rician factor  $K = |m|^2/\gamma^2 = 5$ . Similarly as before, it is seen that error performance improves for sufficiently small duty factors. It is again noted that for  $\nu < 0.8$ ,  $C_q \rightarrow \infty$  as  $P \rightarrow 0$ . Hence, for sufficiently small  $P$ , the zero signal is always declared as the detected signal and the error probability becomes equal to  $\nu$ . Another interesting observation in Fig. 3 is the existence of error floors for sufficiently high values of SNR. This is due to the fact that even if the additive noise vanishes, the performance is limited by the multiplicative noise introduced by unknown fading. Note that in the correct detection probability expressions (43) and (44), the arguments of the Q-function have the term  $P$  in both the numerator and denominator. Hence, letting  $P \rightarrow \infty$  or  $N_0 \rightarrow 0$  does not lead the correct detection probabilities to 1.

## V. CONCLUSION

We have studied the error performance of peaky signaling over fading channels. We have considered two modulation formats: OOBPSK and OQPSK. We have found the optimal

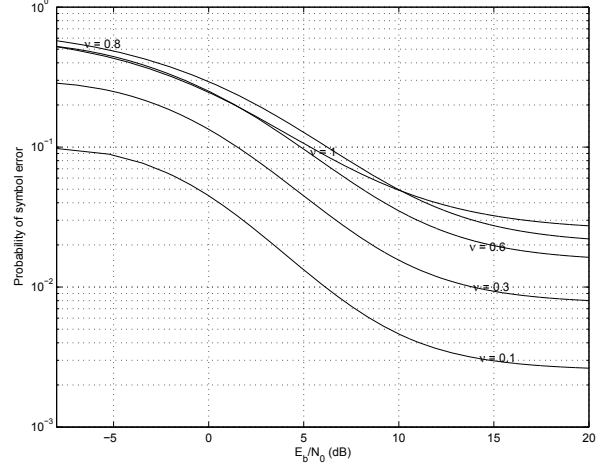


Fig. 3. Probability of symbol error vs.  $E_b/N_0$  for OQPSK signaling with various duty factor values,  $\nu$ , in the noncoherent Rician fading channel with Rician factor  $K = |m|^2/\gamma^2 = 5$ .

MAP decision regions and obtained analytical error probability expressions. Through numerical examples, we have seen that error performance improves if the peakedness of the signaling schemes are sufficiently increased. For fixed error probabilities, substantial gains in terms of SNR per bit are realized, making the peaky signaling schemes energy efficient.

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