Achievable Rates for Pilot-Assisted Transmission over Rayleigh Fading Channels

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***Abstract*— In this paper, transmission over a single-input, single-output, time-varying Rayleigh flat fading channel is considered. Time variations of the channel are modeled by a Gauss-Markov process. It is assumed that neither the transmitter nor the receiver has prior channel state knowledge. The transmitter employs pilot-assisted transmission and periodically sends training symbols for the receiver to estimate the channel. Information rates achieved by binary phase-shift keying modulation are maximized by finding the optimal pilot signal transmission period and optimal training power allocation. Both fast and slow fading conditions are studied. A significant improvement in achievable rates has been shown when training power adaptation is performed. If the power of both training and data symbols are adapted, it is shown that achievable rates can further be increased especially at low SNR values.**

I. Introduction

Increasing demand on fast and reliable wireless communications has led to many studies on the information-theoretic aspects of fading channels, providing the ultimate performance limits (for a survey of recent results, see e.g., [1]). Due to mobility and changing physical environment, wireless channel in general has time-varying characteristics, the knowledge at the receiver and/or transmitter of which has a deep impact on the system performance. Most information-theoretic results are obtained under the assumption of the availability of the perfect channel state information (CSI). The capacity of fading channels when CSI is perfectly known at the receiver was studied initially by Ericson [4] and more recently by Lee [5]. The authors in [6] and [7] investigated the capacity when both the transmitter and receiver perfectly knows the channel. Recently, there has also been results on the other extreme where nobody knows the channel conditions (see e.g., [20], [21] and references therein). However, most practical wireless systems live in between these two extreme cases by employing some form of channel estimation and operating with imperfect channel knowledge. Caire and Shamai [2] studied the channel capacity under general assumptions where CSI at the receiver and transmitter can be noisy versions of the true channel state.

When neither the transmitter nor receiver has prior knowledge of the channel, a common technique to learn the channel conditions is to insert pilot (training) symbols among data symbols. This technique is generally referred to as pilot-assisted transmission (PAT). The insertion of pilot symbols reduce the data rate because they do not carry any information about the data. These pilot signals are known at the receiver and are employed to estimate channel. The GSM system inserts 26 pilot bits placed in the middle of each packet [9] and the North America TDMA standard inserts pilot symbols at the beginning of each packet [10]. Pilot-assisted tranmission is also employed in broadband systems such as HyperLAN II [11] and IEEE 802.11 family [12]-[13]. Wireless broadcast also uses PAT.

There has been a number of studies where pilot-assisted transmission is considered (see e.g., [14], [15], and [16]). Hassibi and Hochwald [17] computed a lower bound on the capacity of block fading multi-input, multi-output channel which is learned by sending training symbols, and maximized this bound by adapting the power and time allocated to training symbols. It is identified that training-based schemes are preferable in high SNR and long coherence time regimes. Abou-Faycal *et al.* [18] studied pilot symbol assisted modulation over Rayleigh fading channels, and considered adaptive coding of data symbols without requiring feedback to the transmitter. It has been shown in [19] that maximum transmission rate can be obtained by inserting pilot symbols periodically. The amount, fraction and placement of the training symbols affects the transmission rate of the system through channel estimation. The more frequently pilot symbols are transmitted, the better the estimation is but the more time is missed for transmitting information. Power allocation among pilot symbols and data symbols is another factor. The more power we use for the training symbols, the better the estimation is but the less power we have for information transfer. An overview of pilot-assisted transmission models, and signal processing and information-theoretic aspects of PAT system design are provided in [8].

In this paper, we consider a time-varying Rayleigh fading channel and assume that no prior channel knowledge is available at the transmitter or receiver. The time variations of the channel conditions are modeled by a Gauss-Markov process. Pilot symbols which are known at the transmitter and receiver are sent with a period of T symbols in order to estimate the channel. The transmitter sends training and data symbols by obeying an average power constraint. We investigate the rates achieved with this particular training scheme.

The rest of the paper is organized as follows. In Section II, we present the channel model. In Section III, we describe the channel estimation method. In Section IV, we give an expression for the achievable rates and formulate the optimization problem. The numerical results are shown in Section V. We

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conclude in Section VI.

II. Channel Model

We consider the following single-input, single-output Rayleigh flat fading channel,

$$Y_k = H_k X_k + N_k \quad k = 1, 2, 3, \ldots \tag{1}$$

where X_k is the complex channel input, Y_k is the complex channel output, H_k and N_k represent the fading and noise components associated with the channel. It is assumed that H_k and N_k are independent zero mean circular complex Gaussian random variables with variances σ_H^2 and σ_N^2, respectively. Moreover, the input X_k is considered to be independent of H_k and N_k.

The fading process is modeled as a first-order Gauss-Markov process:

$$H_k = \alpha H_{k-1} + Z_k \quad 0 \le \alpha \le 1, \quad k = 1, 2, \ldots, \tag{2}$$

where $\{Z_k\}$'s are independent and identically distributed (i.i.d.) circular Gaussian random variables with zero mean and variance (1-α^2)σ_H^2, and α is a parameter that controls the channel variation between any consecutive transmissions. If $\alpha = 1$, the channel stays constant throughout the transmission. If $\alpha = 0$, any consecutive fading coefficients are independent. For bandwidths in the 10kHz range and Doppler spreads of the order of 100 Hz, typical values for α are between 0.9 and 0.99 [18].

III. Pilot-Assisted Transmission

We consider a pilot-assisted transmission scheme where a training signal is transmitted every T symbols in order to estimate the channel fading coefficients, and data is transmitted in between the training signals. The following average power constraint is imposed on the input:

$$\frac{1}{T} \sum_{k=lT}^{(l+1)T-1} E\{|X_k|^2\} \le P, \quad l = 0, 1, 2, \ldots \tag{3}$$

Hence, over a duration of T symbols, the total average power allocated to training and data transmission is limited by PT. In the training mode of the transmission, the channel output is given by

$$Y_{lT} = H_{lT}\sqrt{P_{tr}} + N_{lT} \quad l = 1, 2, 3, \ldots \tag{4}$$

where P_{tr} denotes the power allocated to the pilot signal. At the receiver, minimum mean-square error (MMSE) estimation is employed. Given the observation $Y_{lt} = y_{lt}$, MMSE estimate of H_{lT} is

$$\widehat{H}_{lT} = \frac{\sqrt{P_{tr}}\sigma_H^2}{P_{tr}\sigma_H^2 + \sigma_N^2} y_{lT}. \tag{5}$$

Since a single pilot signal is transmitted for every T symbols, the estimates of the fading coefficients in the data transmission mode are obtained from (5) as follows

$$\widehat{H}_k = \frac{\sqrt{P_{tr}}\sigma_H^2}{P_{tr}\sigma_H^2 + \sigma_N^2}\, \alpha^{k-lT} y_{lT} \quad lT \le k \le (l+1)T - 1. \tag{6}$$

Now, the fading coefficients can be expressed as

$$H_k = \widehat{H}_k + \widetilde{H}_k \tag{7}$$

where $\widetilde{H}_k$ denotes the error in the estimate and has a variance of

$$\sigma_{\tilde{H}_k}^2 = \sigma_H^2 - \frac{P_{tr}\sigma_H^4}{P_{tr}\sigma_H^2 + \sigma_N^2}(\alpha^{k-lT})^2 \quad lT < k < (l+1)T. \tag{8}$$

Therefore, after the estimation, the channel can be represented with the following model:

$$Y_k = \widehat{H}_k X_k + \widetilde{H}_k X_k + N_k \quad k = 1, 2, 3, \ldots \tag{9}$$

We note that a similar pilot-assisted transmission scheme is also studied in [18] where power allocated to training is kept fixed at P. In this paper, we investigate the performance improvements obtained by optimal power allocation among the data and training symbols.

IV. Achievable Rates with Optimal Power Allocation

Since a joint Gaussian setting is considered here, $\widetilde{H}_k$ is a Gaussian random variable conditioned on $Y_{lT} = y_{lT}$, and the channel (9) can be regarded as a noncoherent Rician fading channel where neither the transmitter nor receiver knows $\{\widetilde{H}_k\}$. Abou-Faycal *et al.* [20] studied the noncoherent memoryless Rayleigh fading channels and showed that the capacity-achieving input amplitude distribution is discrete. Not having a closed-form formula, the capacity is computed using numerical techniques. The noncoherent memoryless Rician fading channel is analyzed in [21] where the discreteness of the input is shown when the input is subject to peakedness constraints. OOQPSK signaling, where On-Off keying is overlaid on quadrature phase-shift keying, is shown to be an efficient modulation scheme in the low-power regime. In general, *flash signaling* [3], which has high peak-to-average power ratio, is required in the low-power regime if the channel is only imperfectly known.

In this paper, our focus will be on performance improvements that can be achieved by pilot-assisted transmission even with suboptimal modulation formats. Therefore, we consider a simple binary phase-shift keying (BPSK) modulation (i.e., input distribution consists of two equiprobable points which are located at $\sqrt{P_{data}}$ and $-\sqrt{P_{data}}$). Then, the mutual information between the input and output can be expressed as

$$\begin{aligned} &I_k(X_k; Y_k | Y_{lT} = y_{lT}) = \\ &= \frac{1}{2}\int p_{Y_k|X_k}(y|x_k(1)) \log \frac{p_{Y_k|X_k}(y|x_k(1))}{p_{Y_k}(y)} dy \\ &+ \frac{1}{2}\int p_{Y_k|X_k}(y|x_k(2)) \log \frac{p_{Y_k|X_k}(y|x_k(2))}{p_{Y_k}(y)} dy \end{aligned} \tag{10}$$

where

$$p_{Y_k|X_k}(y_k|x_k) = \frac{1}{\pi(\sigma_{\tilde{H}_k}^2|x_k|^2 + \sigma_N^2)} \exp\left(\frac{-|y_k - \widehat{H}_k x_k|^2}{\sigma_{\tilde{H}_k}^2|x_k|^2 + \sigma_N^2}\right). \tag{11}$$

Over a duration of T symbols, a lower bound to the channel capacity C, and hence an achievable rate is given by

$$E\left[\frac{1}{T}\sum_{k=lT+1}^{(l+1)T-1} I_k(X_k;Y_k|Y_{lT}=y_{lT})\right] \tag{12}$$
$$\leq E\left[\frac{1}{T}\, I(X_{lT+1}^{(l+1)T-1};Y_{lT+1}^{(l+1)T-1}|Y_{lT}=y_{lT})\right] \leq C$$

where the expectation is with respect to Y_{lT}. In the above formulation X_a^b denotes the sequence $\{X_a,\dots,X_b\}$.

Suppose that the total average power is PT (3) over an interval of T symbols. If the power of the training symbol is

$$P_{tr} = \gamma PT \quad 0 \leq \gamma \leq 1, \tag{13}$$

then the remaining power $(1-\gamma)PT$ is equally distributed among $(T-1)$ data symbols. Hence, power per data symbol is

$$P_{data} = \frac{(1-\gamma)PT}{T-1}. \tag{14}$$

In the above formulation, T and γ are the design parameters of the pilot-assisted transmission system. T determines the frequency of the transmission of pilot signals and γ controls the fraction of total power dedicated to training. Note that these parameters also effect the data transmission. For instance, while improving the channel estimate, frequent pilot transmission with high training power reduces the data rate due to relative decrease in the time and power allocated to data transmission. On the other hand, insufficient training will lead to adverse channel conditions under which achievable data rates will also be small.

Here we analyze the tradeoff of these design parameters. For given SNR and α values, our goal is to maximize the achievable rates by finding optimal pilot signal transmission period T^* and optimal training power allocation. Hence, the optimization problem is formulated as follows:

$$(\gamma^*,T^*) = \arg\max_{\substack{0\leq\gamma\leq 1,\\ 1\leq T<\infty}} \frac{1}{T}\sum_{k=lT+1}^{(l+1)T-1} E[I_k(X_k;Y_k|Y_{lT}=y_{lT})]. \tag{15}$$

Since a closed-form solution is unlikely to be feasible, we have employed numerical techniques to solve this optimization.

V. Numerical Results

In this section, the numerical solutions of the optimization problem (15) are summarized for different values of SNR and α.

A. SNR = 0 dB

1) $\alpha = 0.99$: In Figure 1, achievable rates over the Rayleigh fading channel are shown as a function of the training period when SNR is 0 dB and α is 0.99. The curve with solid line corresponds to the achievable rates with optimal training power adaptation γ^*. The dotted curve corresponds to the achievable rates without any power adaptation (i.e., $P_{data} = P_{tr} = P$).

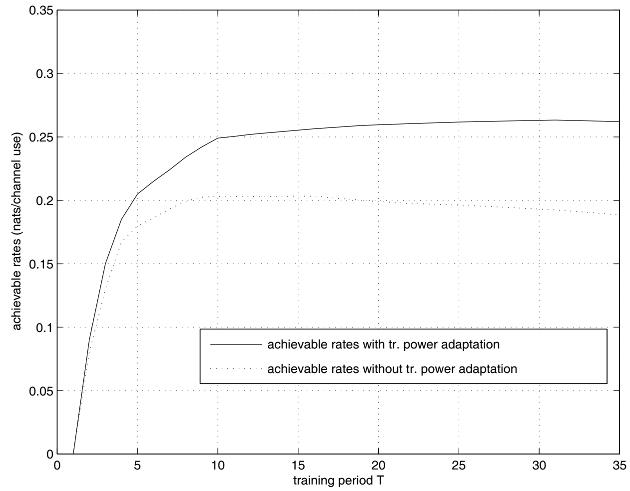

Fig. 1. Comparison with and without training power adaptation, $\alpha = 0.99$ and SNR=0dB

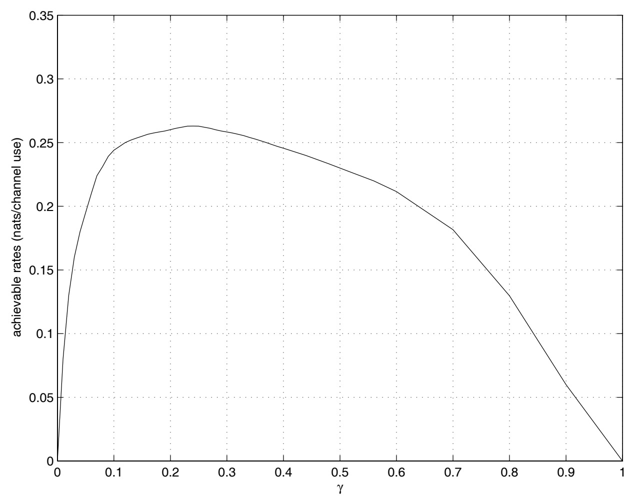

Fig. 2. Achievable rates for different γ values , $\alpha = 0.99$, SNR=0dB and $T = 30$

As T increases up to a certain value, the data rate achieved by using training power adaptation also increases. This is because for higher values of T, we can allocate more power to the training symbols, which in return will give a better channel estimate, without causing a significant decrease in data power. For instance, from equations (13) and (14) for $T = 10$, $P = 1$, and $\gamma = 0.4$, we will have $P_{tr} = 4$, $P_{data} = 0.666$; for $T = 20$, $P = 1$, and $\gamma = 0.4$ we will have $P_{tr} = 8$, $P_{data} = 0.631$.

For $T > 30$, achievable rate curve starts decreasing because of the worsened channel estimation for further data symbols. As seen in Fig. 1, using training power adaptation can yield up to 30% gain compared to the non-adaptive case (when the maximum points are taken into account).

Fig. 2 shows the achievable rates as a function of γ at period

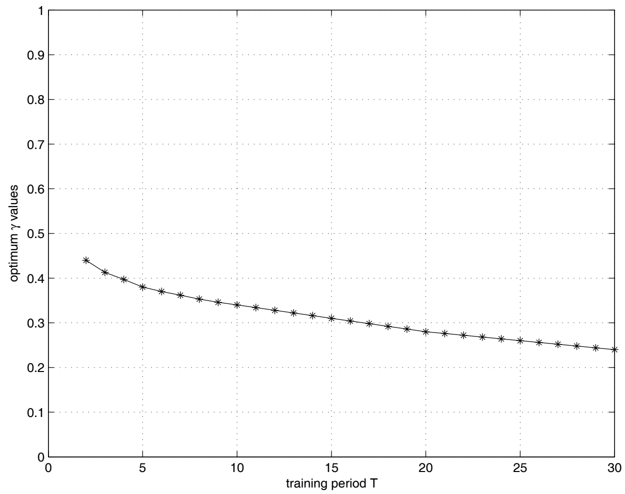

Fig. 3. Optimal γ values for different periods, $\alpha = 0.99$, SNR=$0dB$

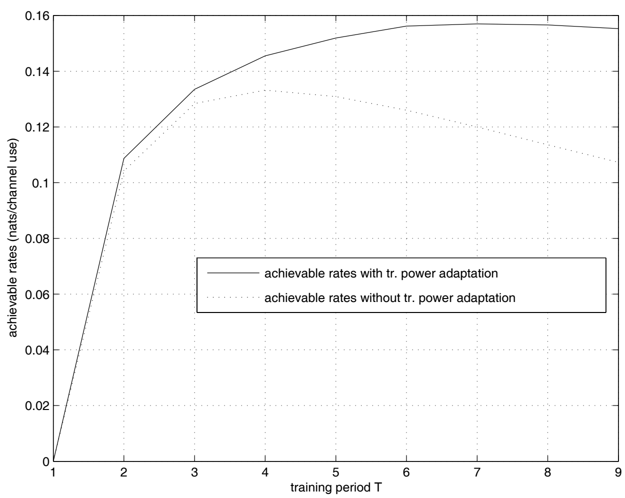

Fig. 4. Comparison with and without training power adaptation, $\alpha = 0.9$ and SNR=$0dB$

$T^* = 30$. Fig. 3 shows the optimal γ values for different T values. As a result, for SNR = 0 dB and $\alpha = 0.99$, transmission with $T^* = 30$ and $\gamma^* = 0.24$ provides the best performance.

2) $\alpha = 0.9$: Achievable rates over the Rayleigh fading channel when SNR is 0 dB and α is 0.9 are plotted in Fig. 4. The curves with optimal training power adaptation and no power adaptation could be seen on the graph.

Since $\alpha = 0.9 < 0.99$, the channel is changing relatively faster and optimal T value is smaller, requiring more frequent pilot symbol transmission. Optimal value of γ is close to 0.4 for $1 < T < 10$. Sending symbols with $T^* = 7$ and $\gamma^* = 0.38$ gives the best performance which has an improvement of 18% over the non-adaptive case.

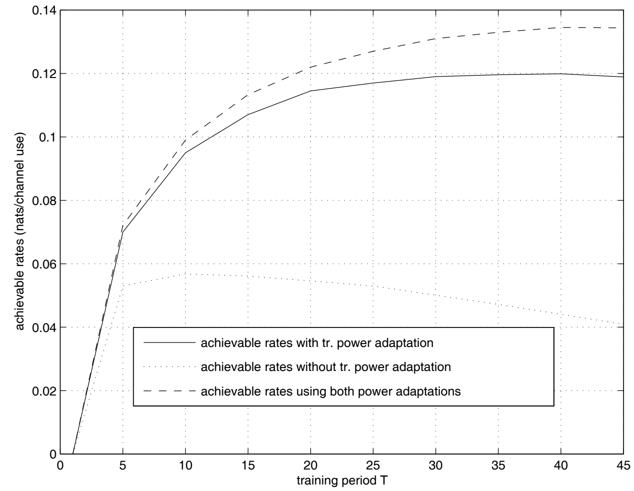

Fig. 5. Comparison of power adaptation techniques, $\alpha = 0.99$ and SNR=$-5dB$

B. SNR = -5 dB

1) $\alpha = 0.99$: In Fig. 5, achievable rates over the Rayleigh fading channel are shown when SNR is -5 dB and α is 0.99. The curve with solid line corresponds to the achievable rates with optimal training power adaptation γ^*. The dotted curve corresponds to the achievable rates without any power adaptation.

As seen in Fig. 5, using training power adaptation can yield up to 110% gain compared to the non-adaptive case.

Moreover, we know that unless the channel is perfectly known, signaling with high peak power is more efficient in the low-SNR regime [3]. Therefore, we can take the advantage of good channel estimation for the first data signals. We can double the data power for the first half of the data signals and do not transmit for the remaining time. For instance, power allocation among data signals will be

$$P_{data_k} = \begin{cases} 2P(1-\gamma)\frac{T}{T-1} & k = lT+1, lT+2, \ldots T\frac{2l+1}{2} \\ 0 & \text{else} \end{cases} \quad (16)$$

when the training period is odd-valued.

When we combine two power adaptation schemes together, the improvement can reach up to 134% as seen in Fig. 5. As a result, for SNR=$-5dB$ and $\alpha = 0.99$, sending symbols with $T^* = 40$ and $\gamma^* = 0.28$ by using both power adaptation policies shows the best performance.

2) $\alpha = 0.9$: Achievable rates over the Rayleigh fading channel when SNR is -5 dB and α is 0.9 are plotted in Fig. 6. Curves with optimal training power adaptation and without any power adaptation could be seen on the graph.

Moreover, as discussed above we can take the advantage of good channel estimation for the first data signals. We can double the data power for the first half of the data signals and do not transmit for the rest time. When we combine the two

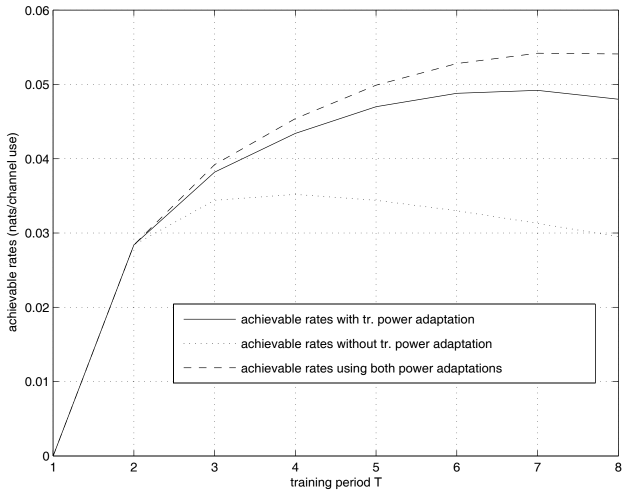

Fig. 6. Comparison of power adaptation techniques, $\alpha = 0.9$ and SNR=$-5dB$

power adaptation schemes, the improvement will be higher as seen in Fig. 6. As a result, for SNR=$-5dB$ and $\alpha = 0.9$, sending symbols with $T^* = 7$ and $\gamma^* = 0.42$ by using both power adaptation policies shows the best performance.

VI. Discussion and Conclusion

We have investigated the achievable rates of the single-input, single-output Rayleigh flat fading channel when there is no prior CSI at the receiver and transmitter and BPSK modulation is employed at the transmitter. We have inserted pilot-symbols among the data symbols in order to estimate channel, then adapted their period and power. For given SNR and α values we have found the highest achievable rates by solving the optimization problem (13) and finding γ^* and T^*. These optimum values provides the best performance. We note that the only feedback requirement to perform training adaptation is that the transmitter knows α values, and hence how fast the channel is changing.

We have shown that adapting the power and period of training symbols as a function of SNR and α values can improve achievable rates significantly. Furthermore, we have adapted the power of the data symbols in low SNR regime to achieve higher gains. Under fast fading conditions, we can not have a sufficiently good channel estimation for a longer periods of time, therefore we have to insert training symbols more frequently which reduces the data rate.

We have employed simple BPSK signaling in this paper. As future work, modulation techniques with higher constellation sizes such as quadrature phase-shift keying (QPSK) or on-off QPSK (OOQPSK) [22] will be considered.

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