The Capacity and Power Efficiency of OOFSK Signaling over Wideband Fading Channels

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***Abstract*— [1] Transmission of information over wideband fading channels using M-ary orthogonal on/off FSK (OOFSK) signaling, in which M-ary FSK signaling is overlaid on on/off keying, is considered. It is assumed that the receiver uses energy detection for the reception of OOFSK signals. Capacity expressions are obtained when the receiver has perfect and imperfect fading side information. Power efficiency is investigated when the transmitter is subject to a peak-to-average power ratio (PAR) limitation or a peak power limitation. It is shown that under PAR limitation, it is extremely power inefficient to operate in the very low SNR regime. On the other hand, if there is only a peak power limitation, it is demonstrated that power efficiency improves as one operates with smaller SNR and vanishing duty factor.**

I. Introduction

The limited availability of energy resources in many wireless communication systems calls for power-efficient transmission schemes. In sensor networks [11], sensor nodes that are densely deployed in a region can only be equipped with a limited power source and in some cases replenishment of these resources may not be possible. Therefore, energy-efficient operation is vital in these systems. Recently, there has also been much interest in ultrawideband systems [12] where low-power pulses of very short duration are used for communication in short distances. These pulses, which require wide bandwidths, must satisfy strict peak power requirements in order not to interfere with existing systems. Although originally proposed as a carrierless time hopping system, ultrawideband radio regulations also allow multicarrier modulation schemes with frequency hopping.

A measure of the power efficiency of digital communication systems is given by the ratio of energy per information bit to noise power per unit bandwidth, $\frac{E_b}{N_0}$. Note that when communicating at rate R bits/s with power P, the transmitted energy per bit is $E_b = \frac{P}{R}$. In the additive white Gaussian noise channel, it is well-known by Shannon channel capacity formula [1] that the minimum bit energy is achieved as the bandwidth grows without bound:

$$\frac{E_b^r}{N_0} = \frac{P/N_0}{B\log_2\left(1+\frac{P}{BN_0}\right)} \underset{B\to\infty}{\longrightarrow} \log_e 2 = -1.59 \text{ dB}. \quad (1)$$

[1] This research was supported by the U.S. Army Research Laboratory under contract DAAD 19-01-2-0011.

This minimum bit energy (1) can be approached by pulse position modulation with vanishing duty cycle [2] or by M-ary orthogonal signaling as M becomes large [3]. Jacobs [4] and Pierce [5] have noticed that even if the transmitted signal undergoes unknown fading, M-ary orthogonal signaling obtained by frequency shift keying (FSK) modulation can still approach (1) for large values of M. More recently, Verdú [6] has proven that the minimum received bit energy of -1.59 dB is achieved in a general class of fading Gaussian channels regardless of the fading knowledge at the receiver and/or transmitter. In particular, it is shown in [6] that if the receiver does not have perfect knowledge of the fading, flash signaling is required to achieve the minimum bit energy. As seen by these results, approaching the minimum bit energy demands not only infinite bandwidth but also, in the case of unknown fading, input signals that are peaky in time or frequency.

It should also be noted that FSK modulation is particularly suitable for noncoherent communications. Butman *et al.* [7] studied the performance of M-ary FSK, which has unit peak-to-average power ratio, over noncoherent Gaussian channels by computing the capacity and computational cut-off rate. Stark [8] analyzed the capacity and cut-off rate of M-ary FSK signaling with both hard and soft decisions in the presence of Rician fading and noted that there exists an optimal code rate for which the required bit energy is minimized. Luo and Médard [10] have shown that FSK with small duty cycle can achieve rates of the order of capacity in ultrawideband systems with limits on bandwidth and peak power.

In this paper, we analyze the capacity and power efficiency of M-ary on/off FSK (OOFSK) signaling in which M-ary FSK signaling is overlaid on on/off keying, enabling us to introduce peakiness in both time and frequency. Motivated by practical considerations, the peakedness of input signals are limited by peak power contraints.

II. Channel Model

In this section, we present the system model. We assume that M-ary orthogonal OOFSK signaling, in which FSK signaling is combined with on-off keying with a fixed duty factor, $\nu \leq 1$, is employed at the transmitter for communication over a fading channel. In this signaling scheme, over the time interval of $[0, T]$ the transmitter either sends no signal with probability

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$1-\nu$, or sends one of M orthogonal sinusoidal signals,

$$s_i(t) = \sqrt{\frac{P}{\nu}}\, e^{j(\omega_i t + \theta_i)} \quad 0 \le t \le T, \quad 1 \le i \le M, \qquad (2)$$

with probability ν. To ensure orthogonality, adjacent frequency slots satisfy $|\omega_{i+1} - \omega_i| = \frac{2\pi}{T}$. Choosing $\nu = 1$, we obtain ordinary FSK signaling. If the channel input is $X = i$ for $1 \le i \le M$, the transmitter sends the sine wave $s_i(t)$, while no transmission is denoted by $X = 0$, and hence $s_0(t) = 0$. Note that OOFSK signaling has average power P, and peak power P/ν. We assume that the transmitted signal undergoes stationary and ergodic fading and that the delay spread of the fading is much less than the symbol duration. We further assume that the symbol duration T is less than the coherence time of the fading. Under these assumptions, the fading has a multiplicative effect on the transmitted signal, and stays constant over the symbol duration. Hence, the received signal can be modeled as follows:

$$r(t) = h_k\, s_{X_k}(t-(k-1)T) + n(t), \; (k-1)T \le t \le kT, \; (3)$$

where $\{X_k\}_{k=1}^{\infty}$ is the input sequence with $X_k \in \{0, 1, 2, \ldots, M\}$, $\{h_k\}_{k=1}^{\infty}$ is a proper complex stationary ergodic fading process with $E\{h_k\} = d$ and $\text{var}(h_k) = \gamma^2$, and $n(t)$ is a zero-mean circularly symmetric complex white Gaussian noise process with single-sided spectral density N_0.

At the receiver, a bank of correlators is employed in each symbol interval to obtain the M-dimensional vector $\mathbf{Y}_k = (Y_{k,1}, \ldots, Y_{k,M})$ where

$$Y_{k,i} = \frac{1}{\sqrt{N_0 T}} \int_{(k-1)T}^{kT} r(t) e^{-j\omega_i t}\, \mathrm{d}t, \quad i = 1, 2, \ldots, M. \; (4)$$

It is easily seen that, given the symbol $X_k = i$, phase θ_i and fading coefficient h_k, $Y_{k,j}$ is a proper complex Gaussian random variable with $E\{Y_{k,j}|X_k = i, \theta_i, h_k\} = \alpha\, h_k\, e^{j\theta_i} \delta_{ij}$ and $\text{var}(Y_{k,j}|X_k = i, \theta_i, h_k) = 1$, where $\delta_{ij} = 1$ if $i = j$ and is zero otherwise, and $\alpha^2 = \frac{PT}{\nu N_0} = \frac{\text{SNR}}{\nu}$ with SNR denoting the signal-to-noise ratio per symbol.

III. Capacity of M-ary Orthogonal OOFSK Signaling with Energy Detection

In this section, we analyze the capacity of M-ary orthogonal OOFSK signaling when in every symbol interval $k = 1, 2, \ldots$, the receiver employs noncoherent reception and measures the energy at each of the M frequencies, i.e., computes for $1 \le i \le M$

$$R_{k,i} = |Y_{k,i}|^2 = \left| \frac{1}{\sqrt{N_0 T}} \int_{(k-1)T}^{kT} r(t) e^{-j\omega_i t}\, \mathrm{d}t \right|^2, \qquad (5)$$

and the decoder sees the vector $\mathbf{R}_k = (R_{k,1}, \ldots, R_{k,M})$. With this structure, the receiver does not need to track phase changes in the channel. We consider the cases where the receiver has either perfect or imperfect fading side information while the transmitter has no knowledge of the fading coefficient. Besides providing the ultimate limits on the rate of communication, capacity results also offer insight into the power efficiency of OOFSK signaling by enabling us to obtain the energy required to send one bit of information reliably.

A. Perfect Receiver Side Information

We first assume that the receiver has perfect knowledge of the magnitude of the fading, $|h|$. For this case, the capacity as a function of $\text{SNR} = \frac{PT}{N_0}$ of M-ary OOFSK signaling with energy detection is given by the following proposition. Throughout the paper, we denote the probability density function and distribution function of a random variable Z by p_Z and F_Z, respectively, with arguments omitted in equations in order to avoid cumbersome expressions.

Proposition 1: Consider the fading channel model (3) and assume that the receiver perfectly knows the instantaneous realization of the magnitude of the fading, $|h_k|, k = 1, 2, \ldots$, while the phase of the fading is unknown. Further assume that the transmitter has no fading side information, and energy detection is performed at the receiver. Then the capacity of M-ary orthogonal OOFSK signaling with a fixed duty factor $\nu \le 1$ is

$$C_M^p(\text{SNR}) = E_{|h|} \left\{ (1-\nu) \int p_{\mathbf{R}|X=0} \log \frac{p_{\mathbf{R}|X=0}}{p_{\mathbf{R}|\,|h|}}\, \mathrm{d}\mathbf{R} + \nu \int p_{\mathbf{R}|X=1,|h|} \log \frac{p_{\mathbf{R}|X=1,|h|}}{p_{\mathbf{R}|\,|h|}}\, \mathrm{d}\mathbf{R} \right\} \qquad (6)$$

where

$$p_{\mathbf{R}|\,|h|} = (1-\nu) p_{\mathbf{R}|X=0} + \frac{\nu}{M} \sum_{i=1}^{M} p_{\mathbf{R}|X=i,|h|}, \qquad (7)$$

$$p_{\mathbf{R}|X=0} = e^{-\sum_{j=1}^{M} R_j}, \qquad (8)$$

$$p_{\mathbf{R}|X=i,|h|} = e^{-\sum_{j=1}^{M} R_j} f(R_i, |h|, \text{SNR}) \quad 1 \le i \le M, \qquad (9)$$

and

$$f(R_i, |h|, \text{SNR}) = e^{-\text{SNR}/\nu\, |h|^2} I_0\left(2\sqrt{\text{SNR}/\nu\, |h|^2 R_i}\right). \qquad (10)$$

Proof: Since the fading is assumed to be a stationary and ergodic process that stays constant over the symbol duration, the capacity of OOFSK signaling can be formulated as follows:

$$C(\text{SNR}) = \lim_{n\to\infty} \max_{X^n} \frac{1}{n} I(X^n; \mathbf{R}^n \,\big|\, |h|^n), \qquad (11)$$

where $X^n = (X_1, \ldots, X_n)$, $\mathbf{R}^n = (\mathbf{R}_1, \ldots, \mathbf{R}_n)$, $|h|^n = (|h_1|, \ldots, |h_n|)$, and $I(\cdot;\cdot)$ denotes the mutual information. As the additive Gaussian noise samples are independent for each symbol interval, it can be shown that an independent and identically distributed (i.i.d.) input sequence achieves the capacity, and due to the symmetry of the channel, the optimal input distribution is uniform over nonzero input values, i.e., $P(X_k = i) = \frac{\nu}{M}$ for $1 \le i \le M$ where $P(X_k = 0) = 1 - \nu$. The capacity expression in (6) is easily obtained by evaluating the mutual information achieved by the optimal input, and considering a generic symbol interval, and dropping the time index k. □

Not having a closed-form formula, the result in (6) must be evaluated numerically, and computational complexity imposes a burden on numerical techniques for large M. Nevertheless, we can find an expression for the channel capacity in the asymptotic regime where $M \uparrow \infty$ as a corollary to Proposition

1. The proof uses martingale theory, following [9] where a similar result is obtained for M-ary FSK signaling over the noncoherent Gaussian channel.

Corollary 1: The capacity expression (6) for M-ary OOFSK signaling in the limit as $M \uparrow \infty$ becomes

$$C_{\infty}^{p}(\text{SNR}) = D(p_{R|\tilde{x},|h|} \,\|\, p_{R|\tilde{x}=0,|h|} \big| F_{|h|}F_{\tilde{x}}) \quad (12)$$

where $D(\cdot \| \cdot \,|\, F_{|h|}F_{\tilde{x}})$ is the Kullback-Leibler divergence conditioned on $|h|$ and $\tilde{x}$, $R = |y|^2 = |h\tilde{x} + n|^2$, $\tilde{x}$ is a two-mass-point discrete random variable with the following distribution,

$$F_{\tilde{x}} = (1-\nu)\, u(\tilde{x}) + \nu\, u(\tilde{x} - \sqrt{\text{SNR}/\nu}), \quad (13)$$

and n is zero-mean circularly symmetric complex Gaussian random variable with $E\{|n|^2\} = 1$. Therefore,

$$p_{R|\tilde{x},|h|} = e^{-R-\tilde{x}^2|h|^2} I_0\left(2\sqrt{\tilde{x}^2\,|h|^2 R}\right).$$

B. Imperfect Receiver Side Information

In this section, we assume that neither the receiver nor the transmitter has any fading side information. Differing from the previous section, here we consider a more special fading process: memoryless Rician fading where each of the i.i.d. h_k's is a proper complex Gaussian random variable with $E\{h_k\} = d$ and $\text{var}(h_k) = \gamma^2$.

Proposition 2: Consider the fading channel (3) and assume that the fading process $\{h_k\}$ is a sequence of i.i.d. proper complex Gaussian random variables which are not known at either the receiver or the transmitter. Further assume that energy detection is performed at the receiver. Then the capacity of M-ary orthogonal OOFSK signaling with fixed duty factor $\nu \le 1$ is given by

$$C_M^{ip}(\text{SNR}) = (1-\nu)\int p_{\mathbf{R}|X=0} \log \frac{p_{\mathbf{R}|X=0}}{p_{\mathbf{R}}}\, d\mathbf{R} + \nu \int p_{\mathbf{R}|X=1} \log \frac{p_{\mathbf{R}|X=1}}{p_{\mathbf{R}}}\, d\mathbf{R} \quad (14)$$

where

$$p_{\mathbf{R}} = (1-\nu)p_{\mathbf{R}|X=0} + \frac{\nu}{M}\sum_{i=1}^{M} p_{\mathbf{R}|X=i}, \quad (15)$$

$$p_{\mathbf{R}|X=0} = e^{-\sum_{j=1}^{M} R_j}, \quad (16)$$

$$p_{\mathbf{R}|X=i} = e^{-\sum_{j=1}^{M} R_j} f(R_i, \text{SNR}) \quad 1 \le i \le M, \quad (17)$$

and

$$f(R_i, \text{SNR}) = \frac{e^{\frac{\frac{\text{SNR}}{\nu}(\gamma^2 R_i - |d|^2)}{\gamma^2\frac{\text{SNR}}{\nu}+1}}}{\gamma^2\frac{\text{SNR}}{\nu}+1} I_0\left(\frac{2\sqrt{\frac{\text{SNR}}{\nu}|d|^2 R_i}}{\gamma^2\frac{\text{SNR}}{\nu}+1}\right). \quad (18)$$

Proof: With the memoryless assumption, the capacity of the M-ary OOFSK signaling can be formulated as the maximum mutual information between the channel input X_k and output vector $\mathbf{R}_k$ for any k. Due to the symmetry of the channel, it can be shown that an input distribution uniform over nonzero input values, i.e., $P(X = i) = \frac{\nu}{M}$ for $1 \le i \le M$ where $P(X = 0) = 1 - \nu$ achieves the capacity and we easily obtain (14) by noting that conditioned on $X = i$, $R_j = |Y_j|^2$ is a chi-square random variable with two degrees of freedom. □

Similarly to Corollary 1, we can find the infinite bandwidth capacity achieved as the number of orthogonal frequencies increases without bound.

Corollary 2: The capacity expression (14) of M-ary OOFSK signaling in the limit as $M \uparrow \infty$ becomes

$$C_{\infty}^{ip}(\text{SNR}) = D(p_{R|\tilde{x}} \,\|\, p_{R|\tilde{x}=0} \big| F_{\tilde{x}}) \quad (19)$$

where $D(\cdot \| \cdot | F_{\tilde{x}})$ denotes the Kullback-Leibler divergence conditioned on $\tilde{x}$, $R = |y|^2 = |h\tilde{x}+n|^2$, $\tilde{x}$ is a two-mass-point discrete random variable with the distribution function given in (13), and n is a zero-mean circularly symmetric complex Gaussian random variable with $E\{|n|^2\} = 1$. Therefore,

$$p_{R|\tilde{x}} = \frac{1}{\gamma^2\tilde{x}^2+1}\exp\left(-\frac{R+\tilde{x}^2|d|^2}{\gamma^2\tilde{x}^2+1}\right) I_0\left(\frac{2\sqrt{\tilde{x}^2|d|^2 R}}{\gamma^2\tilde{x}^2+1}\right).$$

Remark 1: Assume that in the case of perfect receiver side information, $\{h_k\}$ is a sequence of i.i.d. proper complex Gaussian random variables. Then the asymptotic loss in capacity incurred by not knowing the fading is

$$C_{\infty}^{p}(\text{SNR}) - C_{\infty}^{ip}(\text{SNR}) = I(|h|; R \,|\, \tilde{x}) \quad (20)$$

where $R = |h\tilde{x} + n|^2$.

Remark 2: Consider the case of imperfect receiver side information where

$$\begin{aligned} C_{\infty}^{ip} &= D(p_{R|\tilde{x}} \,\|\, p_{R|\tilde{x}=0} \big| F_{\tilde{x}}) \\ &= (\gamma^2 + |d|^2)\text{SNR} - \nu \log\left(\gamma^2\frac{\text{SNR}}{\nu}+1\right) - \frac{2\text{SNR}|d|^2}{\gamma^2\text{SNR}/\nu+1} \\ &\quad + \nu E_R\left\{\log I_0\left(\frac{2\sqrt{\frac{\text{SNR}}{\nu}|d|^2 R}}{\gamma^2\frac{\text{SNR}}{\nu}+1}\right)\right\} \end{aligned} \quad (21)$$

with $\text{SNR} = \frac{PT}{N_0}$. From (21) we can easily see that for fixed symbol interval T,

$$\lim_{\nu \downarrow 0} \frac{1}{T} C_{\infty}^{ip}(\text{SNR}) = (\gamma^2 + |d|^2)\frac{P}{N_0} \text{ nats/s}, \quad (22)$$

and for fixed duty factor ν,

$$\lim_{T \uparrow \infty} \frac{1}{T} C_{\infty}^{ip}(\text{SNR}) = (\gamma^2 + |d|^2)\frac{P}{N_0} \text{ nats/s}. \quad (23)$$

Note that right-hand sides of (22) and (23) are equal to the infinite bandwidth capacity of the unfaded Gaussian channel with the same received signal power. Hence, these results agree with previous results [4], [5] and [13] where it has been shown that the capacity of M-ary FSK signaling over noncoherent fading channels approaches the infinite bandwidth capacity of the unfaded Gaussian channel for large M and large symbol duration T or small duty factor ν.

IV. Power Efficiency

In this section, we analyze the power efficiency of OOFSK signaling by studying the energy per information bit requirement in the low-SNR regime. In the low-power regime, the spectral-efficiency/bit-energy tradeoff reflects the fundamental tradeoff between bandwidth and power. Assuming that the bandwidth of M-ary OOFSK modulation is $\frac{M}{T}$ where T is the symbol duration, the maximum achievable spectral efficiency is

$$\mathsf{C}\left(\frac{E_b}{N_0}\right) = \frac{1}{M}\, C(\textsc{snr}) \quad \text{bits/s/Hz} \tag{24}$$

where $C(\textsc{snr})$ is the capacity in bits/symbol, and

$$\frac{E_b}{N_0} = \frac{\textsc{snr}}{C(\textsc{snr})} \tag{25}$$

is the bit energy normalized to the noise power. For average power limited channels, the bit energy required for reliable communications decreases monotonically with decreasing spectral efficiency, and the minimum bit energy is achieved at zero spectral efficiency, i.e., $\frac{E_b}{N_0}_{\min} = \lim_{\text{SNR}\to 0} \frac{\text{SNR}}{C(\text{SNR})} \log_e 2 = \frac{\log_e 2}{\dot{C}(0)}$. Hence for fixed rate transmission, reduction in the required power comes only at the expense of increased bandwidth, and the minimum bit energy is achieved only in the asymptotic regime of infinite bandwidth. If one is willing to spend more power, then reliable communication over a finite bandwidth is possible. Hence achieving the minimum bit energy is not a sufficient criterion for finite bandwidth analysis. Verdú [6] has recently given the following formula for the wideband slope, defined as the slope of the spectral efficiency curve $\mathsf{C}\left(\frac{E_b}{N_0}\right)$ in bits/s/Hz/3dB at zero spectral efficiency:

$$S_0 \stackrel{\text{def}}{=} \lim_{\frac{E_b}{N_0}\downarrow \frac{E_b}{N_0}\big|_{\mathsf{C}=0}} \frac{\mathsf{C}\left(\frac{E_b}{N_0}\right)}{10\log_{10}\frac{E_b}{N_0} - 10\log_{10}\frac{E_b}{N_0}\Big|_{\mathsf{C}=0}}\, 10\log_{10} 2$$

$$= \frac{1}{M}\frac{2\left(\dot{C}(0)\right)^2}{-\ddot{C}(0)}, \tag{26}$$

where $\dot{C}(0)$ and $\ddot{C}(0)$ denote the first and second derivatives of the capacity in nats. The wideband slope closely approximates the growth of the spectral efficiency curve in the low-power regime and hence is a useful tool providing insightful results when bandwidth is a resource to be conserved.

A. Limited Peak-to-Average Power Ratio

The peak-to-average power ratio (PAR) of OOFSK signaling is equal to inverse of the duty factor, $1/\nu$. In this section, we keep the duty factor fixed at its minimum allowed level while the $\textsc{snr}$ varies. We show that under this limited PAR condition, OOFSK communication with energy detection at low $\textsc{snr}$ values is extremely power inefficient even in the unfaded Gaussian channel.

Proposition 3: The first derivative of the capacity at zero $\textsc{snr}$ achieved by M-ary OOFSK signaling with a fixed duty factor $\nu \leq 1$ over the unfaded Gaussian channel is zero, i.e., $\dot{C}(0) = 0$, and hence the bit energy required at zero spectral efficiency is infinite,

$$\frac{E_b}{N_0}\bigg|_{\mathsf{C}=0} = \lim_{\textsc{snr}\to 0} \frac{\textsc{snr}}{C(\textsc{snr})} \log_e 2 = \frac{\log_e 2}{\dot{C}(0)} = \infty. \tag{27}$$

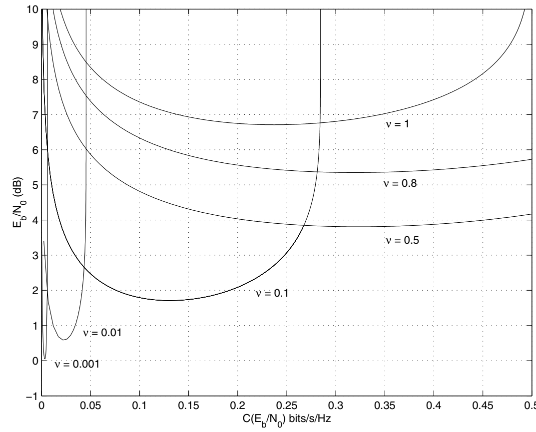

Fig. 1. $\frac{E_b}{N_0}$ (dB) vs. $\mathsf{C}\left(\frac{E_b}{N_0}\right)$ bits/s/Hz for the unfaded Gaussian channel. $M = 2$.

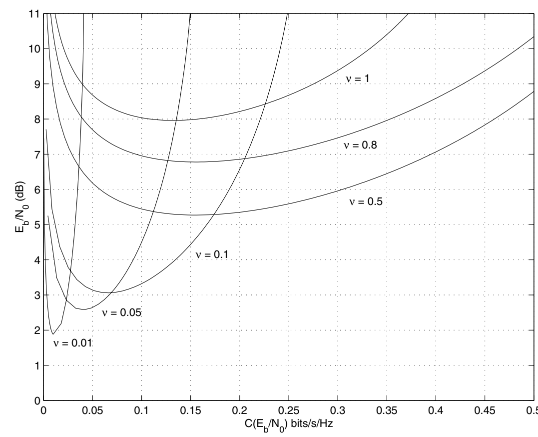

Fig. 2. $\frac{E_b}{N_0}$ (dB) vs. $\mathsf{C}\left(\frac{E_b}{N_0}\right)$ bits/s/Hz for the Rician channel with $\mathsf{K} = 0.5$. $M = 2$.

Since the presence of fading unknown at the transmitter does not increase the capacity, from the above Proposition, we immediately conclude that $\dot{C}(0) = 0$ for fading channels regardless of receiver side information as long as ν is fixed and hence the peak-to-average power ratio is limited. This result shows that the minimum bit energy of M-ary OOFSK signaling is achieved at a nonzero spectral efficiency, C^*, and operating at spectral efficiencies lower than C^* (hence operating at very low $\textsc{snr}$ values) has to be avoided as this only increases the required power. Note that since $\textsc{snr} = \frac{PT}{N_0}$, low $\textsc{snr}$ means either low input power P with respect to the noise power, or small symbol duration T which requires high bandwidth. We need to resort to numerical computations to obtain the most power-efficient operating points.

Figure 1 plots bit energy curves as a function of the spectral efficiency in bits/s/Hz achieved in the unfaded Gaussian channel by 2-OOFSK signaling for different values of fixed duty factor ν. Notice that for all cases minimum bit energy values are obtained at a nonzero spectral efficiency and as the duty factor is decreased, the required minimum bit energy is also decreased. With $\nu = 0.001$, the minimum bit energy is about 0.05 dB. Note that this is a significant improvement over the case $\nu = 1$ (ordinary FSK) where the minimum bit energy is about 6.7 dB. However, this gain is obtained at the cost of a very large increase in the peak-to-average ratio. Fig. 2 plots the bit energy curves in the Rician channel with Rician factor $\mathsf{K} = \frac{|d|^2}{\gamma^2} = 0.5$.

B. Limited Peak Power

In this section, we consider the case where the peak level of the transmitted signal is limited while there is no constraint on the peak-to-average power ratio. Hence we fix the peak level to the maximum allowed level, $A = \frac{P}{\nu}$. Therefore as $P \to 0$, the duty factor also has to vanish and hence the peak-to-average ratio increases without bound. In this case, the minimum bit energy is achieved at zero spectral efficiency and the wideband slope provides a good characterization of the bandwith/power tradeoff at low spectral efficiency values.

Proposition 4: Assume that the transmitter is limited in peak power, $\frac{P}{\nu} \leq A$, and the symbol duration T is fixed. Then the capacity achieved by M-ary OOFSK signaling, with fixed peak power A, is a concave function of the SNR. For the perfect receiver side information case the minimum received bit energy and the wideband slope are

$$\frac{E_b^r}{N_0}_{\min} = \frac{\log_e 2}{\frac{E_{|h|}E_R\{\log I_0(2\sqrt{\eta|h|^2R})\}}{\eta(\gamma^2+|d|^2)} - 1},$$

$$S_0 = \frac{2\left(E_h E_R\left\{\log I_0\left(2\sqrt{\eta|h|^2R}\right)\right\} - \eta(\gamma^2+|d|^2)\right)^2}{E_h\{I_0(2\eta|h|^2)\} - 1},$$

respectively, where R is a noncentral chi-square random variable with $p_R = e^{-R-\eta|h|^2} I_0(2\sqrt{\eta|h|^2R})$ and $\eta = A\frac{T}{N_0}$ is the normalized peak power. For the imperfect receiver side information case the minimum received bit energy and the wideband slope are

$$\frac{E_b^r}{N_0}_{\min} = \log_e 2\left(1 - \frac{1}{\gamma^2+|d|^2}\left(\frac{2|d|^2}{\eta\gamma^2+1} + \frac{\log(\eta\gamma^2+1) - E\left\{\log I_0\left(\frac{2\sqrt{\eta|d|^2R}}{\eta\gamma^2+1}\right)\right\}}{\eta}\right)\right)^{-1},$$

$$S_0 = \begin{cases} \dfrac{2\left(\eta(\gamma^2+|d|^2) - \frac{2\eta|d|^2}{\eta\gamma^2+1} - \log(\eta\gamma^2+1) + E\left\{\log I_0\left(\frac{2\sqrt{\eta|d|^2R}}{\eta\gamma^2+1}\right)\right\}\right)^2}{\frac{1}{1-\eta^2\gamma^4}\exp\left(\frac{2\eta^2\gamma^2|d|^2}{1-\eta^2\gamma^4}\right) I_0\left(\frac{2\eta|d|^2}{1-\eta^2\gamma^4}\right) - 1} & \eta\gamma^2 < 1 \\ 0 & \eta\gamma^2 \geq 1, \end{cases}$$

respectively, where R is a noncentral chi-square random variable with $p_R = \frac{1}{\eta\gamma^2+1}\exp\left(-\frac{R+\eta|d|^2}{\eta\gamma^2+1}\right) I_0\left(\frac{2\sqrt{\eta|d|^2R}}{\eta\gamma^2+1}\right)$.

In contrast to the limited PAR case, the minimum bit energy is achieved at zero spectral efficiency, and hence the power efficiency of the system improves if one operates at lower SNR and hence duty factor. Note that for both cases, the minimum bit energy and the wideband slope do not depend on M. Therefore on/off signaling with vanishing duty cycle is optimally power-efficient at very low spectral efficiency values, and there is no need for frequency modulation. Further note that in the imperfect receiver side information case, if $\eta\gamma^2 \geq 1$, then $S_0 = 0$, and hence approaching the minimum bit energy is extremely slow. If we relax the peak power limitation and let $\eta \uparrow \infty$, then it is easily seen that even in the imperfect receiver side information case, $\frac{E_b^r}{N_0}_{\min} \to \log_e 2 = -1.59$ dB. Indeed, Verdú [6] has shown in a more general setting that flash signaling with increasingly high peak power is required to achieve the minimum bit energy of -1.59 dB if the fading is not perfectly known.

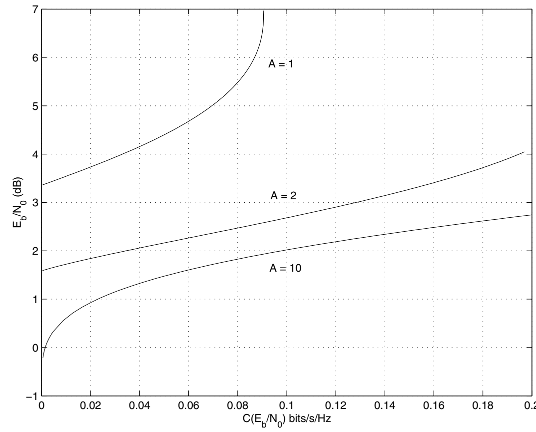

Fig. 3. $\frac{E_b}{N_0}$ (dB) vs. $\mathsf{C}\left(\frac{E_b}{N_0}\right)$ bits/s/Hz for the unfaded Gaussian channel. $M = 2$.

Fig. 3 plots the bit energy curves achieved by 2-OOFSK signaling in the unfaded Gaussian channel for different peak power values A. Notice that for all cases the minimum bit energy is achieved at zero spectral efficiency or equivalently as SNR $\to 0$, and this energy monotonically decreases to -1.59 dB as $A \to \infty$.

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