# Error Rate Analysis for Peaky Signaling over Fading Channels

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***Abstract*—In this paper, the performance of signaling strategies with high peak-to-average power ratio is analyzed in both coherent and noncoherent fading channels. Two modulation schemes, namely on-off phase-shift keying (OOPSK) and on-off frequency-shift keying (OOFSK), are considered. The optimal detector structures are identified and analytical expressions for the error probabilities are obtained for arbitrary constellation sizes. Numerical techniques are employed to compute the error rates. It is concluded that increasing the peakedness of the signals results in reduced error rates for a given power level and hence equivalently improves the energy efficiency for fixed error probabilities.**

***Index Terms*—Peaky signaling, maximum a-posteriori probability (MAP) detection, error probability, on-off keying, phase-shift keying, frequency-shift keying.**

## I. Introduction

In wireless communications, when the receiver and transmitter have only imperfect knowledge of the channel conditions, efficient transmission strategies have a markedly different structure than those employed over perfectly known channels. For instance, Abou-Faycal *et al.* [1] studied the noncoherent Rayleigh fading channel where the receiver and transmitter has no channel side information, and showed that the capacity-achieving input amplitude distribution is discrete with a finite number of mass points. It has also been shown that there always exists a mass point at the origin. These results indicate that unlike perfectly known channels where a Gaussian input is optimal and a continuum of amplitude levels are available for transmission, only finitely many amplitude levels with one level at the origin should be used for transmission over noncoherent Rayleigh channels. The discreteness of the optimal amplitude distribution has also been shown over other noncoherent channels (see e.g., [2], [3], [4], [6], and references therein).

Another key result for noncoherent channels is the requirement of transmission with high peak-to-average power ratio in the low signal-to-noise ratio (SNR) regime. In noncoherent Rayleigh channels [1], the capacity-achieving amplitude distribution has two mass points for low SNR values, and the nonzero mass migrates away from the origin, increasing the peak power, as SNR decreases. Indeed, this behavior has been shown in a more general setting in [5] where *flash signaling* is proven to be the necessary form of transmission to achieve the capacity of imperfectly-known fading channels in the low-SNR regime. The impact upon the channel capacity of using signals with limited peakedness is investigated in [7], [8], [9], and [10]. Recently, the error probability of coded systems in noncoherent fading channels are studied in [11]–[16] through the analysis of the error exponents and reliability function.

In this paper, we consider two particular peaky modulation schemes: on-off phase-shift keying (OOPSK) and on-off frequency-shift keying (OOFSK). We determine the optimal detector structures and analyze the error probabilities of uncoded OOPSK and OOFSK. The analysis is conducted for both coherent and noncoherent fading channels, and error performances are investigated at low-to-high SNR levels. We find that the error performance of signaling with high peak-to-average power ratio is superior to that of conventional PSK and FSK modulations over the entire SNR range if the duty cycles of modulations are small enough. Equivalently, peaky signaling is shown to be more energy efficient for fixed error probabilities. As a result of the analysis conducted in this paper, information-theoretic inspired signaling schemes, OOPSK and OOFSK, emerge as energy efficient modulation formats especially well-suited for low data rate applications such as in sensor networks.

The rest of the paper is organized as follows. We describe the modulation techniques and the channel model in Section II. We analyze the error performance in fading channels in Section III. Finally, Section IV includes our conclusions.

## II. System Model

In this section, we present the system model. We consider two types of signaling at the transmitter side, OOPSK and OOFSK. Basically, these two modulation schemes are obtained by overlaying on-off keying on phase-shift keying and frequency-shift keying, respectively. In both signaling schemes, transmitter sends over the symbol interval of $[0, T]$ either no signal with probability $1-\nu$ or one of $M$ signals, each with probability $\nu/M$. The transmitted signal, for $0 \le t \le T$, can be mathematically expressed as

$$s_i(t) = \begin{cases} \sqrt{\frac{P}{\nu}}\, e^{j(\omega_i t + \theta_i)} & 1 \le i \le M \quad \text{with prob. } \nu/M \\ 0 & i = 0 \quad \text{with prob. } (1-\nu) \end{cases} \tag{1}$$

where $P$ is the average power, $\omega_i$ and $\theta_i$ are the frequency and phase of $s_i(t)$, respectively. $s_0(t) = 0$ denotes no transmission. In OOPSK modulation, the frequency is fixed, i.e., $\omega_i = \omega$ $\forall i$, and phases are $\theta_i = \frac{2\pi(i-1)}{M}$ for $1 \le i \le M$. In OOFSK, information is carried by the frequencies and each nonzero signal has a distinct frequency. To ensure orthogonality, adjacent frequency slots satisfy $|\omega_{i+1} - \omega_i| = \frac{2\pi}{T}$. In OOFSK,

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phases can be arbitrary. In both modulations, $\nu$ can be regarded as the duty cycle of the transmission. Note also that both modulation formats have an average power of $P$ and a peak power of $\frac{P}{\nu}$, and hence a peak-to-average power ratio (PAR) of $\frac{1}{\nu}$. Limitations on the PAR of the signaling scheme, which may be dictated by regulations or system component specifications, are reflected in the choice of the value of $\nu$.

We assume that the transmitted signal undergoes stationary and ergodic fading and that the delay spread of the fading is much less than the symbol duration. Under this narrowband assumption, the fading has a multiplicative effect on the transmitted signal. If we further assume that the symbol duration $T$ is less than the coherence time of the fading, then the fading stays constant over the symbol duration. Hence, if the transmitted signal is $s_i(t)$, the received signal is

$$r(t) = h_k\, s_i(t-(k-1)T) + n(t), \text{ for } \begin{array}{l} (k-1)T \le t \le kT \\ i = 0,1,2,\ldots,M \\ k = 1,2,\ldots \end{array}$$

where $\{h_k\}$ denotes the sequence of fading coefficients and is a proper, complex, stationary, ergodic fading process with finite variance, and $n(t)$ is a zero-mean, circularly symmetric, white complex Gaussian noise process with single-sided spectral density $N_0$. The transmitted signal, fading coefficients, and additive noise are assumed to be mutually independent of each other.

If OOPSK modulation is used at the transmitter, the receiver demodulates the received signal using a correlator:

$$y_k = \frac{1}{\sqrt{N_0 T}} \int_{(k-1)T}^{kT} r(t) e^{-j\omega(t-(k-1)T)} dt = \begin{cases} \alpha h_k e^{j\theta_i} + n_k & 1 \le i \le M \\ n_k & i = 0 \end{cases} \quad k = 1,2,3\ldots \tag{2}$$

where $\alpha = \sqrt{\frac{PT}{\nu N_0}} = \sqrt{\frac{\text{SNR}}{\nu}}$, $\theta_i = \frac{2\pi(i-1)}{M}$ for $1 \le i \le M$, and $\{n_k\}$ is a sequence of independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero mean and variance $E\{|n_k|^2\} = 1$.

If OOFSK signals are transmitted, a bank of $M$ correlators is employed at the receiver and the output of the $m^{th}$ correlator at time $t = kT$ for $k = 1,2,\ldots,$ is given by:

$$y_{k,m} = \frac{1}{\sqrt{N_0 T}} \int_{(k-1)T}^{kT} r(t) e^{-j\omega_m(t-(k-1)T)} dt = \begin{cases} \alpha h_k e^{j\theta_i} + n_{k,m} & m = i \\ n_{k,m} & m \ne i \end{cases}, \text{ for } \begin{array}{l} m = 1,2,\ldots,M \\ i = 0,1,\ldots,M \end{array}, \tag{3}$$

where again $\alpha = \sqrt{\frac{PT}{\nu N_0}}$, and $\{n_{k,m}\}$ is an i.i.d sequence in both $k$ and $m$ of zero-mean unit-variance circularly symmetric complex Gaussian random variables. The output of $M$ demodulators is denoted by the $M$-dimensional vector $\mathbf{y}_k = [y_{k,1}, y_{k,2}, \ldots, y_{k,M}]$.

## III. Error Probability in Fading Channels

In this section, we analyze the error performance of OOPSK and OOFSK in fading channels. For the detection of these signals, maximum a posteriori probability (MAP) decision rule, which minimizes the probability of error, is employed after demodulation at the receiver. It is assumed that symbol-by-symbol detection is performed. We initially consider noncoherent Rician fading channels in which neither the transmitter nor the receiver knows the instantaneous realizations of the fading coefficients. In this case, $h_k$ for all $k$ is a proper complex Gaussian random variable with mean $E\{h_k\} = d$ and variance $E\{|h_k - d|^2\} = \gamma^2$. It is assumed that the channel statistics and hence the values of $d$ and $\gamma^2$ are assumed to be known at the transmitter and receiver. Results obtained for noncoherent Rician fading channels are subsequently specialized to coherent fading channels using the following approach: If the receiver has perfect knowledge of the channel fading coefficients $\{h_k\}$, the conditional mean and variance values are $E\{h_k|h_k\} = h_k$ and $E\{|h_k - E\{h_k|h_k\}|^2|h_k\} = 0$, respectively. Therefore, if we assume $\gamma^2 = 0$ and replace $d$ by the random gain $h_k$ in the results of noncoherent fading channels, we obtain the performance results for the coherent fading channel model where only the receiver has perfect knowledge of the channel fading coefficients $\{h_k\}$. Note that in this case, the distribution of $\{h_k\}$ can be arbitrary.

### A. OOPSK

In this section, OOPSK signaling is considered. We first derive the results for noncoherent Rician fading channels. Since symbol-by-symbol detection is employed, we henceforth drop the time index $k$ without loss of generality. In MAP detection, $s_i$ is the detected signal if

$$p_i f_{y|s_i}(y|s_i) > p_j f_{y|s_j}(y|s_j) \quad \forall j \ne i \tag{4}$$

where $p_i$ and $p_j$ denote the prior transmission probabilities of the signals $s_i$ and $s_j$, respectively, and the conditional probability density function in the absence of receiver channel knowledge is given by

$$f_{y|s_i}(y|s_i) = \begin{cases} \frac{1}{\pi(\alpha^2\gamma^2+1)} e^{-\frac{|y-\alpha d e^{j\theta_i}|^2}{\alpha^2\gamma^2+1}} & 1 \le i \le M \\ \frac{1}{\pi} e^{-|y|^2} & i = 0 \end{cases}. \tag{5}$$

Note from (1) that $p_i = \nu/M$ for $i \ne 0$ and $p_0 = 1-\nu$. The following proposition describes the optimal decision regions and provides an expression for the error probability of OOPSK signaling in noncoherent Rician fading channels. Note that the results immediately specialize to noncoherent Rayleigh fading channels when it is assumed that $d = 0$. The proofs of the results presented in this paper are relegated to [20] which is the extended version of this paper.

*Proposition 1:* The optimal decision regions for OOPSK signals when transmitted over noncoherent Rician fading channels are

$$\mathcal{D}_i = \left\{ y = |y|e^{j\theta_y} : \frac{2\pi(i-\frac{3}{2})}{M} \le \theta_y \le \frac{2\pi(i-\frac{1}{2})}{M} \right. \text{ and } \left. \alpha\gamma^2|y|^2 + 2|d|\,|y|\cos(\theta_y - \theta_i) > \tau \right\} \quad 1 \le i \le M, \tag{6}$$

and

$$\mathcal{D}_0 = \left\{ y = |y|e^{j\theta_y} : \alpha\gamma^2|y|^2 + 2|d|\,|y|\cos(\theta_y - \theta_i) < \tau \quad \forall i \ne 0 \right\} \tag{7}$$

where

$$\tau = \begin{cases} \zeta & \zeta \geq 0 \\ 0 & \zeta < 0 \end{cases} \tag{8}$$

and

$$\zeta = \alpha|d|^2 + \left(\alpha\gamma^2 + \frac{1}{\alpha}\right) \ln\left(\frac{M(1-\nu)}{\nu}\left(1+\alpha^2\gamma^2\right)\right). \tag{9}$$

Furthermore, the error probability is given by

$$P_e = 1 - ((1-\nu)P_{c|s_0} + \nu P_{c|s_1}) \tag{10}$$

where

$$P_{c|s_0} = M\int_0^{\hat{\tau}}\left(1 - 2Q\left(\sqrt{2}x\tan\frac{\pi}{M}\right)\right)\frac{1}{\sqrt{\pi}}e^{-x^2}\,dx - M\int_{\hat{\tau}}^{\tilde{\tau}}\left(1 - 2Q\left(\sqrt{2}\sqrt{\frac{\tau}{\alpha\gamma^2} - x^2 - \frac{2|d|}{\alpha\gamma^2}x}\right)\right)\frac{1}{\sqrt{\pi}}e^{-x^2}\,dx, \tag{11}$$

and

$$P_{c|s_1} = \int_{\hat{\tau}}^{\infty}\left(1 - 2Q\left(\frac{\sqrt{2}\,x\tan\frac{\pi}{M}}{\sqrt{1+\alpha^2\gamma^2}}\right)\right) \times \frac{1}{\sqrt{\pi(1+\alpha^2\gamma^2)}}e^{-\frac{(x-\alpha|d|)^2}{1+\alpha^2\gamma^2}}\,dx - \int_{\hat{\tau}}^{\tilde{\tau}}\left(1 - 2Q\left(\frac{\sqrt{2}\sqrt{\frac{\tau}{\alpha\gamma^2} - x^2 - \frac{2|d|}{\alpha\gamma^2}x}}{\sqrt{1+\alpha^2\gamma^2}}\right)\right) \times \frac{1}{\sqrt{\pi(1+\alpha^2\gamma^2)}}e^{-\frac{(x-\alpha|d|)^2}{1+\alpha^2\gamma^2}}\,dx \tag{12}$$

are the correct detection probabilities when $s_0(t)$ and $s_1(t)$, respectively, are the transmitted signals. In the above integral expressions,

$$\hat{\tau} = \frac{1}{1+\tan^2\frac{\pi}{M}}\left(\sqrt{\frac{|d|^2}{\alpha^2\gamma^4} + \frac{\tau}{\alpha\gamma^2}\left(1+\tan^2\frac{\pi}{M}\right)} - \frac{|d|}{\alpha\gamma^2}\right)$$

and

$$\tilde{\tau} = \sqrt{\frac{|d|^2}{\alpha^2\gamma^4} + \frac{\tau}{\alpha\gamma^2}} - \frac{|d|}{\alpha\gamma^2}.$$

□

The following corollary to Proposition 1 provides the decision regions and the error probability of OOPSK modulation in coherent fading channels.

*Corollary 1:* In coherent fading channels, the decision regions as a function of the instantaneous realization of fading magnitude, $|h|$, are given by (6) and (7) when we assume $\gamma^2 = 0$ and replace $d$ by $h$. Moreover, the error probability is

$$P_{e|\,|h|} = 1 - \left((1-\nu)P_{c|s_0,|h|} + \nu P_{c|s_1,|h|}\right) \tag{13}$$

where

$$P_{c|s_0,|h|} = M\int_0^{\frac{\tau}{2|h|}}\left(1 - 2Q\left(\sqrt{2}x\tan\frac{\pi}{M}\right)\right)\frac{e^{-x^2}}{\sqrt{\pi}}\,dx \tag{14}$$

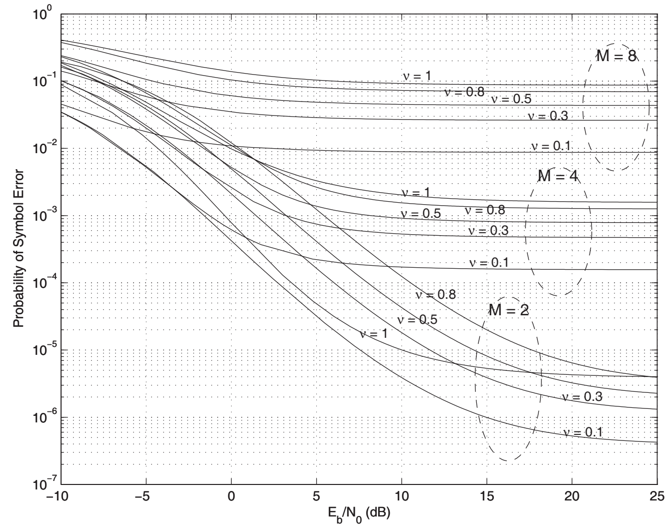


Fig. 1. Probability of symbol error vs. $E_b/N_0$ for OOPSK signaling with duty factor values $\nu = 1, 0.8, 0.5, 0.3, 0.1$ and constellation sizes $M = 2, 4, 8$ in the noncoherent Rician fading channel with Rician factor $\mathsf{K} = |d|^2/\gamma^2 = 10$.

and

$$P_{c|s_1,|h|} = \int_{\frac{\tau}{2|h|}}^{\infty}\left(1 - 2Q\left(\sqrt{2}\,x\tan\frac{\pi}{M}\right)\right)\frac{e^{-(x-\alpha|h|)^2}}{\sqrt{\pi}}\,dx \tag{15}$$

where $\tau$ is obtained by again assuming $\gamma^2 = 0$ and $d = h$ in (8). The probability of error averaged over the realizations of the fading magnitude is obtained from

$$P_e = \int_0^{\infty} P_{e|\,|h|}\,dF_{|h|}(|h|) \tag{16}$$

where $F_{|h|}$ is the distribution function of the fading magnitude. □

It has been shown in [20] that if $\nu < \frac{M}{M+1}$, $P_{c|s_0} \to 1$, $P_{c|s_1} \to 0$ as SNR vanishes, and therefore $P_e \to \nu$ (and similarly $P_{e||h|} \to \nu$ in coherent fading channels). On the other hand, as SNR $\to \infty$ and hence $\alpha \to \infty$, we have $\lim_{\text{SNR}\to\infty} P_{c|s_0} = 1$ and

$$\lim_{\text{SNR}\to\infty} P_{c|s_1} = \int_{-\frac{|d|}{\gamma}}^{\infty}\left(1 - 2Q\left(\sqrt{2}\tan\frac{\pi}{M}\left(\hat{x} + \frac{|d|}{\gamma}\right)\right)\right)\frac{1}{\sqrt{\pi}}e^{-\hat{x}^2}\,d\hat{x} \stackrel{\text{def}}{=} P_{c,\infty|s_1}. \tag{17}$$

Note that $P_{c|s_1}$ does not approach to 1 as SNR $\to \infty$. Hence, we experience an error floor in noncoherent channels.

Fig. 1 plots the error probability curves for OOPSK signaling for constellation sizes of $M = 2, 4,$ and 8 in the noncoherent Rician fading channel with Rician factor $\mathsf{K} = |d|^2/\gamma^2 = 10$. Note that in ordinary $M$-PSK modulation, each symbol carries $\log_2 M$ bits. It is important to note that in $M$-OOPSK modulation, the maximum number of bits that can be carried by each symbol is equal to the entropy $H(\nu) = \nu\log_2(M/\nu) + (1-\nu)\log_2(1/(1-\nu))$ which decreases to zero as $\nu \to 0$. Hence, decreasing the duty cycle diminishes

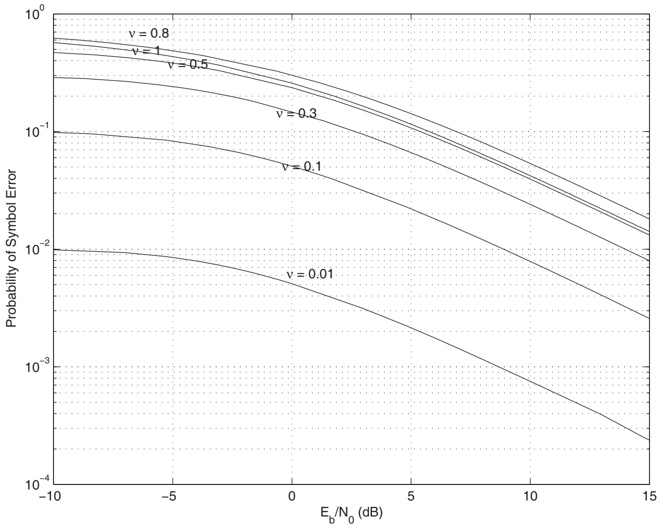


Fig. 2. Probability of symbol error vs. $E_b/N_0$ for 4-OOPSK signaling with different duty factor values, $\nu$, in coherent Rayleigh fading channels with $E\{|h|^2\} = 1$.

the data rates. Since information rate is at most equal to the entropy, bit assignments to the signals can be done so that the signals are encoded at the entropy rate. Since the signals are not equiprobable, variable-length bit assignment is required. For instance, such a bit assignment can be accomplished using Huffman coding [19]. For fair comparison, Fig. 1 plots the curves as a function of the SNR normalized by the entropy of the $M$-OOPSK source, giving the SNR per bit. It is seen in all cases that the error performance improves as duty factor value decreases sufficiently. Note that for 8-OOPSK, even having a duty value of $\nu = 0.8$ improves the performance with respect to the regular 8-PSK in the entire range of SNR per bit values considered in the graph. On the other hand, when $M = 2$, decreasing the duty cycle to $\nu = 0.8, 0.5$ or 0.3 does not provide gains with respect to the case of $\nu = 1$ unless SNR is high enough. As predicted, we observe error floors in all cases. We note that error floors decrease with decreasing constellation sizes and duty factors.

Figure 2 plots the average error probability curves for 4-OOPSK signaling with different duty cycle values in the coherent Rayleigh fading channel with $E\{|h|^2\} = 1$. It is again observed from the figure that if the peakedness of the input signals is increased sufficiently (e.g., $\nu = 0.3, 0.1, 0.01$), significant improvements in error performance are achieved over the ordinary PSK (i.e., OOPSK with $\nu = 1$) performance. We note that 4-OOPSK with $\nu = 0.8$ performs worse than regular 4-PSK. As discussed above, for $\nu < \frac{M}{M+1} = 0.8$, error probabilities approach $\nu$ as SNR $\to 0$.

## B. OOFSK

In this section, we study OOFSK signaling and again initially consider noncoherent Rician channels. The output of the bank of $M$ demodulators is $\mathbf{y} = (y_1, y_2, \ldots, y_M)$. If $s_i(t)$ is transmitted, then $y_m$ is a proper complex Gaussian random variable with mean $E\{y_m|s_i\} = \alpha d e^{j\theta_i}\delta_{mi}$ and variance $var\{y_m|s_i\} = \alpha^2\gamma^2\delta_{mi} + 1$ where $\delta_{mi} = 1$ if $m = i$ and zero otherwise. We assume that energy detection is employed. Hence, the detector observes $\mathbf{R} = (R_1, R_2, \ldots, R_M)$ where $R_m = |y_m|^2$ which gives the energy in the $m^{\text{th}}$ frequency. Since $y_m$ is a complex Gaussian random variable, $R_m = |y_m|^2$ is chi-square distributed and the joint distribution function of the output vector $\mathbf{R}$ conditioned on $s_i(t)$ being transmitted is

$$f_{\mathbf{R}|s_i}(\mathbf{R}|s_i) = \begin{cases} e^{-\sum_{\substack{j=1\\ j\neq i}}^{M} R_j} \dfrac{e^{-\frac{R_i+\alpha^2|d|^2}{1+\alpha^2\gamma^2}}}{1+\alpha^2\gamma^2} I_0\left(\dfrac{2\sqrt{R_i\alpha^2|d|^2}}{1+\alpha^2\gamma^2}\right) & 1 \le i \le M \\ e^{-\sum_{j=1}^{M} R_j}, & i = 0 \end{cases}$$

where we have used the fact that $\{R_j\}$ are independent random variables conditioned on $s_i$. The following result provides the optimal detection rule and the error probability of OOFSK signaling in noncoherent Rician fading channels. Similarly as before, the proof can be found in [20].

*Proposition 2:* The optimal MAP detection rule for OOFSK signaling over noncoherent Rician fading channels is given as follows: $s_i(t)$ for $i \neq 0$ is the detected signal if

$$R_i > R_j \quad \forall j \neq i \quad \text{and} \quad R_i > \tau = \begin{cases} \Phi^{-1}(\xi) & \xi \geq 1 \\ 0 & \xi < 1 \end{cases} \tag{18}$$

where

$$\Phi(x) = e^{\frac{\alpha^2\gamma^2 x}{1+\alpha^2\gamma^2}} I_0\left(\frac{2\sqrt{x\,\alpha^2|d|^2}}{1+\alpha^2\gamma^2}\right) \tag{19}$$

and $\xi = \frac{M(1-\nu)}{\nu}(1 + \alpha^2\gamma^2)\, e^{\frac{\alpha^2|d|^2}{1+\alpha^2\gamma^2}}$. $s_0(t)$ is the detected signal if $R_i < \tau \;\forall i$. The probability of error is

$$P_e = 1 - ((1-\nu)P_{c|s_0} + \nu P_{c|s_1}) \tag{20}$$

where $P_{c|s_0} = (1 - e^{-\tau})^M$, and

$$P_{c|s_1} = \sum_{n=0}^{M-1} (-1)^n \binom{M-1}{n} \frac{e^{-\frac{n\alpha^2|d|^2}{n(1+\alpha^2\gamma^2)+1}}}{n(1+\alpha^2\gamma^2)+1} \times Q_1\left(\sqrt{\frac{2\alpha^2|d|^2}{(1+\alpha^2\gamma^2)[n(1+\alpha^2\gamma^2)+1]}}, \sqrt{\frac{2[n(1+\alpha^2\gamma^2)+1]\tau}{(1+\alpha^2\gamma^2)}}\right) \tag{21}$$

are the correct detection probabilities when $s_0(t)$ and $s_1(t)$, respectively, are the transmitted signals. In the above formulation, $Q_1(\cdot,\cdot)$ denotes the Marcum $Q$-function [18]. □

*Corollary 2:* In coherent fading channels, the decision rules and the error probability of OOFSK as a function of the instantaneous realization of fading magnitude, $|h|$, are immediately obtained by assuming $\gamma^2 = 0$ and replacing $d$ by $h$ in the formulas given in Proposition 2.

Fig. 3 provides the error rates of 16-OOFSK over the noncoherent Rayleigh fading channel. We observe that unlike the OOPSK case, OOFSK performance is free of error floors at high SNRs. This fact is proved in [20]. As SNR decreases, we see that $P_e \to \nu$ for the cases in which $\nu < M/(M+1)$. Moreover, we note that modulations with $\nu < 1$ perform better than those with $\nu = 1$ at low SNRs. However, at high SNR levels, gains are realized if the duty factor is sufficiently small. Figure 4 plots the average probability of error values of 16-OOFSK as a function of SNR per bit for different values of duty

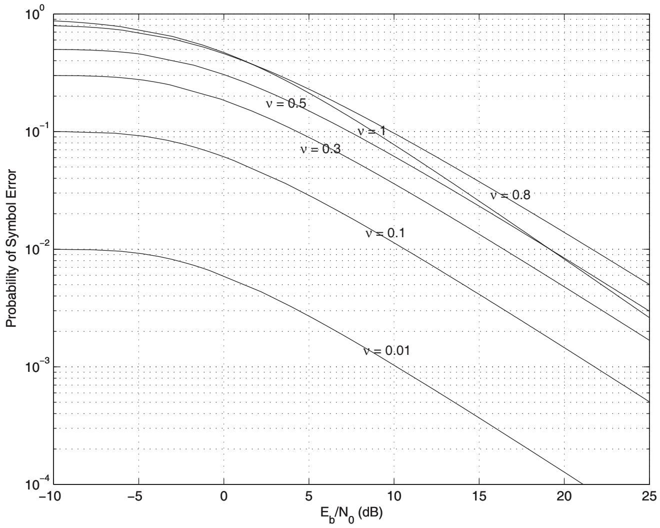


Fig. 3. Error probability of 16-OOFSK in the noncoherent Rayleigh fading channel with duty factors $\nu = 1, 0.8, 0.5, 0.3, 0.1$, and $0.01$.

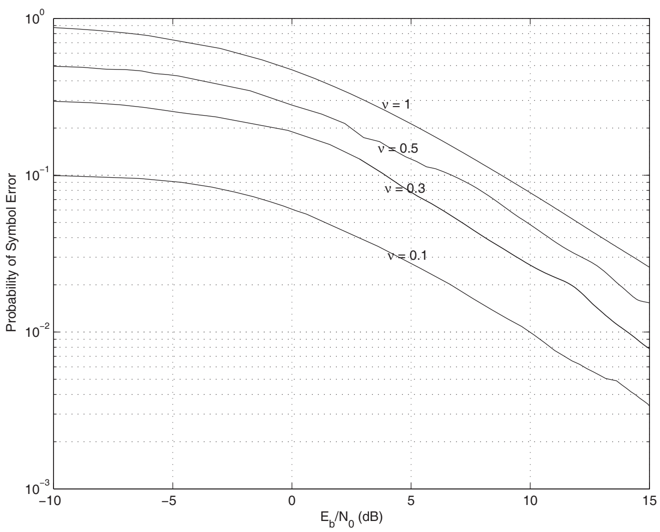


Fig. 4. Probability of error of 16-OOFSK in the coherent Rayleigh fading channel with $E\{|h|^2\} = 1$. The duty factor values are $\nu = 1, , 0.5, 0.3$ and $0.1$.

cycle parameter $\nu$ in the coherent Rayleigh fading channel with $E\{|h|^2\} = 1$. Similarly, OOFSK signaling with low duty cycle has superior performance in terms of error rates. From another perspective, if the duty cycle of the modulation is reduced, the same performance can be achieved at smaller SNR per bit values, improving the energy efficiency.

## IV. Conclusion

We have studied the error performance of peaky signaling over fading channels. We have considered two modulation formats: OOPSK and OOFSK. We have found the optimal MAP decision rules and obtained analytical error probability expressions for OOPSK and OOFSK transmissions over both coherent and noncoherent fading channels. Through numerical results, we have seen that the error performance improves if the peakedness of the signaling schemes are sufficiently increased. For fixed error probabilities, substantial gains in terms of SNR per bit are realized, making the peaky signaling schemes energy efficient. Since decreasing the duty cycle diminishes the communication rates, information-theoretic inspired OOPSK and OOFSK emerge as energy-efficient modulation techniques well-suited for low data rate applications.

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