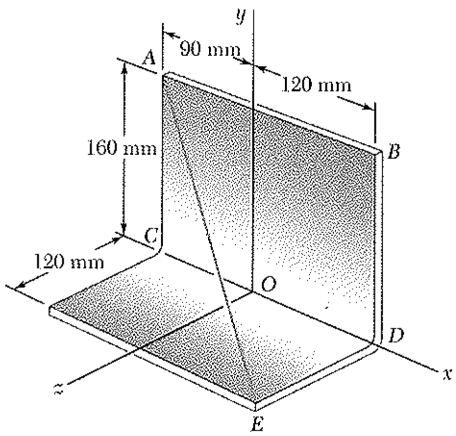
Homework #4 Solution



PROBLEM 3.21

The wire AE is stretched between the corners A and E of a bent plate. Knowing that the tension in the wire is 435 N, determine the moment about O of the force exerted by the wire (*a*) on corner A, (*b*) on corner E.

SOLUTION

$$\overline{AE} = (0.21 \text{ m})\mathbf{i} - (0.16 \text{ m})\mathbf{j} + (0.12 \text{ m})\mathbf{k}$$

$$AE = \sqrt{(0.21 \text{ m})^2 + (-0.16 \text{ m})^2 + (0.12 \text{ m})^2} = 0.29 \text{ m}$$

(*a*) $$\mathbf{F}_A = F_A \boldsymbol{\lambda}_{AE} = F\frac{\overline{AE}}{AE}$$

$$= (435 \text{ N})\frac{0.21\mathbf{i} - 0.16\mathbf{j} + 0.12\mathbf{k}}{0.29}$$

$$= (315 \text{ N})\mathbf{i} - (240 \text{ N})\mathbf{j} + (180 \text{ N})\mathbf{k}$$

$$\mathbf{r}_{A/O} = -(0.09 \text{ m})\mathbf{i} + (0.16 \text{ m})\mathbf{j}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.09 & 0.16 & 0 \\ 315 & -240 & 180 \end{vmatrix}$$

$$= 28.8\mathbf{i} + 16.20\mathbf{j} + (21.6 - 50.4)\mathbf{k}$$

$$\mathbf{M}_O = (28.8 \text{ N}\cdot\text{m})\mathbf{i} + (16.20 \text{ N}\cdot\text{m})\mathbf{j} - (28.8 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

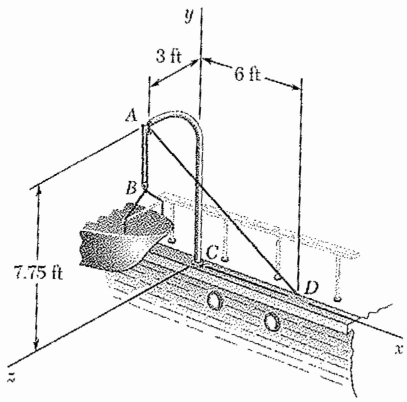
(*b*) $$\mathbf{F}_E = -\mathbf{F}_A = -(315 \text{ N})\mathbf{i} + (240 \text{ N})\mathbf{j} - (180 \text{ N})\mathbf{k}$$

$$\mathbf{r}_{E/O} = (0.12 \text{ m})\mathbf{i} + (0.12 \text{ m})\mathbf{k}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.12 & 0 & 0.12 \\ -315 & 240 & -180 \end{vmatrix}$$

$$= -28.8\mathbf{i} + (-37.8 + 21.6)\mathbf{j} + 28.8\mathbf{k}$$

$$\mathbf{M}_O = -(28.8 \text{ N}\cdot\text{m})\mathbf{i} - (16.20 \text{ N}\cdot\text{m})\mathbf{j} + (28.8 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.22

A small boat hangs from two davits, one of which is shown in the figure. The tension in line *ABAD* is 82 lb. Determine the moment about *C* of the resultant force $\mathbf{R}_A$ exerted on the davit at *A*.

SOLUTION

We have $$\mathbf{R}_A = 2\mathbf{F}_{AB} + \mathbf{F}_{AD}$$

where $$\mathbf{F}_{AB} = -(82 \text{ lb})\mathbf{j}$$

and $$\mathbf{F}_{AD} = \mathbf{F}_{AD}\frac{\overrightarrow{AD}}{AD} = (82 \text{ lb})\frac{6\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k}}{10.25}$$

$$\mathbf{F}_{AD} = (48 \text{ lb})\mathbf{i} - (62 \text{ lb})\mathbf{j} - (24 \text{ lb})\mathbf{k}$$

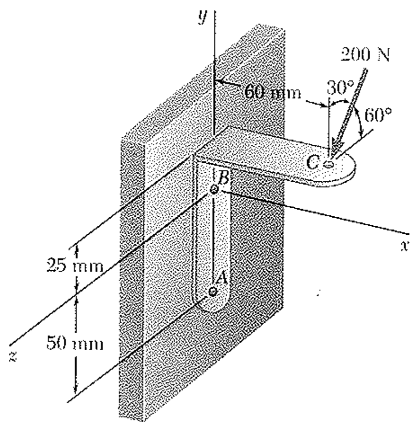
Thus $$\mathbf{R}_A = 2\mathbf{F}_{AB} + \mathbf{F}_{AD} = (48 \text{ lb})\mathbf{i} - (226 \text{ lb})\mathbf{j} - (24 \text{ lb})\mathbf{k}$$

Also $$\mathbf{r}_{A/C} = (7.75 \text{ ft})\mathbf{j} + (3 \text{ ft})\mathbf{k}$$

Using Eq. (3.21): $$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 7.75 & 3 \\ 48 & -226 & -24 \end{vmatrix}$$

$$= (492 \text{ lb}\cdot\text{ft})\mathbf{i} + (144.0 \text{ lb}\cdot\text{ft})\mathbf{j} - (372 \text{ lb}\cdot\text{ft})\mathbf{k}$$

$$\mathbf{M}_C = (492 \text{ lb}\cdot\text{ft})\mathbf{i} + (144.0 \text{ lb}\cdot\text{ft})\mathbf{j} - (372 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.25

A 200-N force is applied as shown to the bracket *ABC*. Determine the moment of the force about *A*.

SOLUTION

We have $\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}_C$

where $\mathbf{r}_{C/A} = (0.06 \text{ m})\mathbf{i} + (0.075 \text{ m})\mathbf{j}$

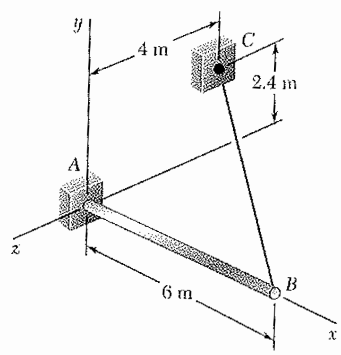
$$\mathbf{F}_C = -(200 \text{ N})\cos 30°\mathbf{j} + (200 \text{ N})\sin 30°\mathbf{k}$$

Then

$$\mathbf{M}_A = 200\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.06 & 0.075 & 0 \\ 0 & -\cos 30° & \sin 30° \end{vmatrix}$$

$$= 200[(0.075\sin 30°)\mathbf{i} - (0.06\sin 30°)\mathbf{j} - (0.06\cos 30°)\mathbf{k}]$$

or $\mathbf{M}_A = (7.50 \text{ N}\cdot\text{m})\mathbf{i} - (6.00 \text{ N}\cdot\text{m})\mathbf{j} - (10.39 \text{ N}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.26

The 6-m boom *AB* has a fixed end *A*. A steel cable is stretched from the free end *B* of the boom to a Point *C* located on the vertical wall. If the tension in the cable is 2.5 kN, determine the moment about *A* of the force exerted by the cable at *B*.

SOLUTION

First note

$$d_{BC} = \sqrt{(-6)^2 + (2.4)^2 + (-4)^2}$$
$$= 7.6 \text{ m}$$

Then

$$\mathbf{T}_{BC} = \frac{2.5 \text{ kN}}{7.6}(-6\mathbf{i} + 2.4\mathbf{j} - 4\mathbf{k})$$

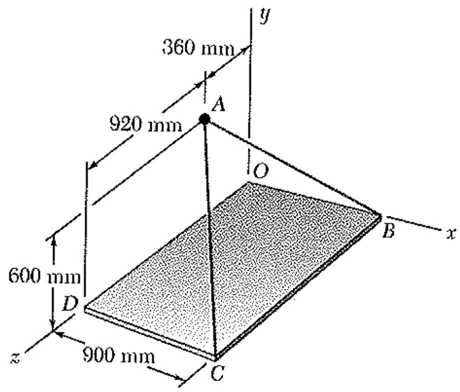
We have $\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BC}$

where $\mathbf{r}_{B/A} = (6 \text{ m})\mathbf{i}$

Then

$$\mathbf{M}_A = (6 \text{ m})\mathbf{i} \times \frac{2.5 \text{ kN}}{7.6}(-6\mathbf{i} + 2.4\mathbf{j} - 4\mathbf{k})$$

or $\mathbf{M}_A = (7.89 \text{ kN}\cdot\text{m})\mathbf{j} + (4.74 \text{ kN}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.47

Knowing that the tension in cable AB is 570 N, determine the moment about each of the coordinate axes of the force exerted on the plate at B.

SOLUTION

$$\overline{BA} = (-900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}$$

$$BA = \sqrt{(-900)^2 + (600)^2 + (360)^2} = 1140 \text{ mm}$$

$$\mathbf{F}_B = F_B \frac{\overline{BA}}{BA}$$

$$= (570 \text{ N})\frac{-900\mathbf{i} + 600\mathbf{j} + 360\mathbf{k}}{1140}$$

$$= -(450 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} + (180 \text{ N})\mathbf{k}$$

$$\mathbf{r}_B = (0.9 \text{ m})\mathbf{i}$$

$$\mathbf{M}_O = \mathbf{r}_B \times \mathbf{F}_B = 0.9\mathbf{i} \times (-450\mathbf{i} + 300\mathbf{j} + 180\mathbf{k})$$

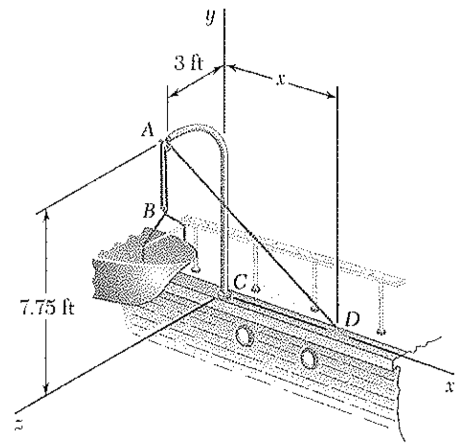
$$= 270\mathbf{k} - 162\mathbf{j}$$

$$\mathbf{M}_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

$$= -(162 \text{ N}\cdot\text{m})\mathbf{j} + (270 \text{ N}\cdot\text{m})\mathbf{k}$$

Therefore,

$$M_x = 0, \; M_y = -162.0 \text{ N}\cdot\text{m}, \; M_z = +270 \text{ N}\cdot\text{m} \blacktriangleleft$$



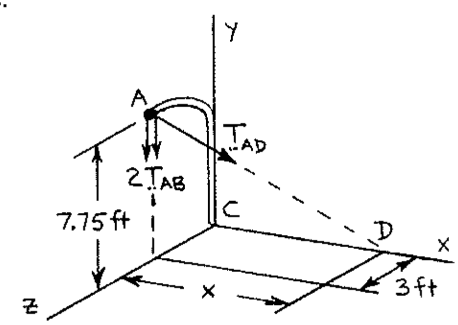
PROBLEM 3.49

A small boat hangs from two davits, one of which is shown in the figure. It is known that the moment about the z-axis of the resultant force $\mathbf{R}_A$ exerted on the davit at A must not exceed 279 lb·ft in absolute value. Determine the largest allowable tension in line $ABAD$ when $x = 6$ ft.

SOLUTION

First note: $\mathbf{R}_A = 2\mathbf{T}_{AB} + \mathbf{T}_{AD}$

Also note that only $\mathbf{T}_{AD}$ will contribute to the moment about the z-axis.



Now
$$AD = \sqrt{(6)^2 + (-7.75)^2 + (-3)^2}$$
$$= 10.25 \text{ ft}$$

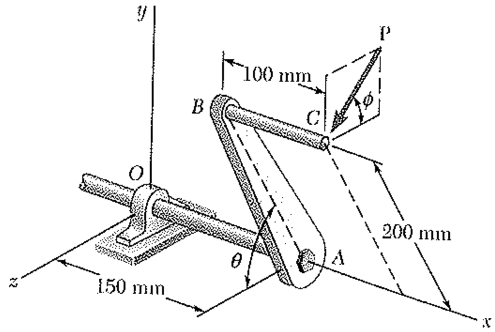
Then
$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD}$$
$$= \frac{T}{10.25}(6\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k})$$

Now
$$M_z = \mathbf{k}\cdot(\mathbf{r}_{A/C} \times \mathbf{T}_{AD})$$

where
$$\mathbf{r}_{A/C} = (7.75 \text{ ft})\mathbf{j} + (3 \text{ ft})\mathbf{k}$$

Then for T_{max},
$$279 = \left|\frac{T_{max}}{10.25}\begin{vmatrix} 0 & 0 & 1 \\ 0 & 7.75 & 3 \\ 6 & -7.75 & -3 \end{vmatrix}\right|$$
$$= \frac{T_{max}}{10.25}\left|-(1)(7.75)(6)\right|$$

or $T_{max} = 61.5$ lb ◀



PROBLEM 3.53

A single force **P** acts at C in a direction perpendicular to the handle BC of the crank shown. Knowing that $M_x = +20$ N · m and $M_y = -8.75$ N · m, and $M_z = -30$ N · m, determine the magnitude of **P** and the values of ϕ and θ.

SOLUTION

$$\mathbf{r}_C = (0.25 \text{ m})\mathbf{i} + (0.2 \text{ m})\sin\theta\mathbf{j} + (0.2 \text{ m})\cos\theta\mathbf{k}$$

$$\mathbf{P} = -P\sin\phi\mathbf{j} + P\cos\phi\mathbf{k}$$

$$\mathbf{M}_O = \mathbf{r}_C \times \mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.25 & 0.2\sin\theta & 0.2\cos\theta \\ 0 & -P\sin\phi & P\cos\phi \end{vmatrix}$$

Expanding the determinant, we find

$$M_x = (0.2)P(\sin\theta\cos\phi + \cos\theta\sin\phi)$$

$$M_x = (0.2)P\sin(\theta + \phi) \qquad (1)$$

$$M_y = -(0.25)P\cos\phi \qquad (2)$$

$$M_z = -(0.25)P\sin\phi \qquad (3)$$

Dividing Eq. (3) by Eq. (2) gives: $\tan\phi = \dfrac{M_z}{M_y}$ (4)

$$\tan\phi = \frac{-30 \text{ N}\cdot\text{m}}{-8.75 \text{ N}\cdot\text{m}}$$

$$\phi = 73.740$$

$\phi = 73.7°$ ◀

Squaring Eqs. (2) and (3) and adding gives:

$$M_y^2 + M_z^2 = (0.25)^2 P^2 \quad \text{or} \quad P = 4\sqrt{M_y^2 + M_z^2} \qquad (5)$$

$$P = 4\sqrt{(8.75)^2 + (30)^2}$$

$$= 125.0 \text{ N}$$

$P = 125.0$ N ◀

Substituting data into Eq. (1):

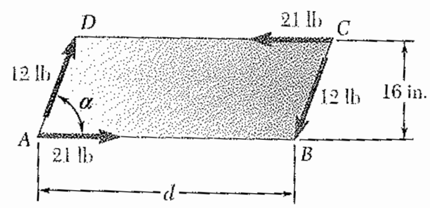
$$(+20 \text{ N}\cdot\text{m}) = 0.2 \text{ m}(125.0 \text{ N})\sin(\theta + \phi)$$

$$(\theta + \phi) = 53.130° \quad \text{and} \quad (\theta + \phi) = 126.87°$$

$$\theta = -20.6° \quad \text{and} \quad \theta = 53.1°$$

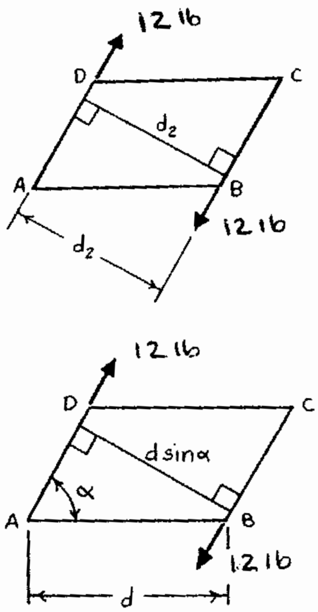
$Q = 53.1°$ ◀

PROBLEM 3.70



A plate in the shape of a parallelogram is acted upon by two couples. Determine (*a*) the moment of the couple formed by the two 21-lb forces, (*b*) the perpendicular distance between the 12-lb forces if the resultant of the two couples is zero, (*c*) the value of α if the resultant couple is 72 lb·in. clockwise and *d* is 42 in.

SOLUTION



(*a*) We have $M_1 = d_1 F_1$

where $d_1 = 16$ in.

$F_1 = 21$ lb

$M_1 = (16 \text{ in.})(21 \text{ lb})$

$= 336 \text{ lb·in.}$ or $\mathbf{M}_1 = 336 \text{ lb·in.}$ ↺ ◀

(*b*) We have $\mathbf{M}_1 + \mathbf{M}_2 = 0$

or $336 \text{ lb·in.} - d_2(12 \text{ lb}) = 0$ $d_2 = 28.0$ in. ◀

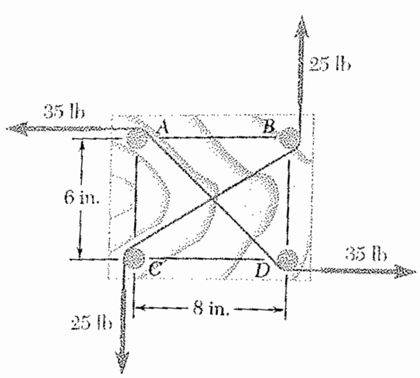
(*c*) We have $\mathbf{M}_{\text{total}} = \mathbf{M}_1 + \mathbf{M}_2$

or $-72 \text{ lb·in.} = 336 \text{ lb·in.} - (42 \text{ in.})(\sin\alpha)(12 \text{ lb})$

$\sin\alpha = 0.80952$

and $\alpha = 54.049°$ or $\alpha = 54.0°$ ◀

PROBLEM 3.72

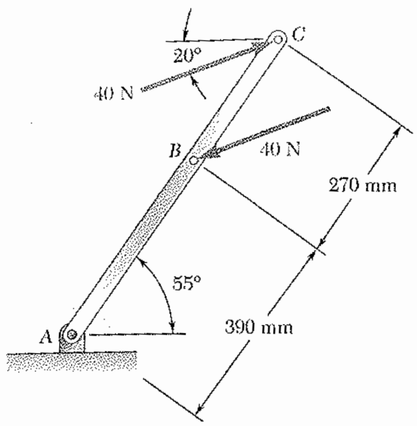


Four pegs of the same diameter are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. Determine the diameter of the pegs knowing that the resultant couple applied to the board is 485 lb·in. counterclockwise.

SOLUTION

$$M = d_{AD}F_{AD} + d_{BC}F_{BC}$$

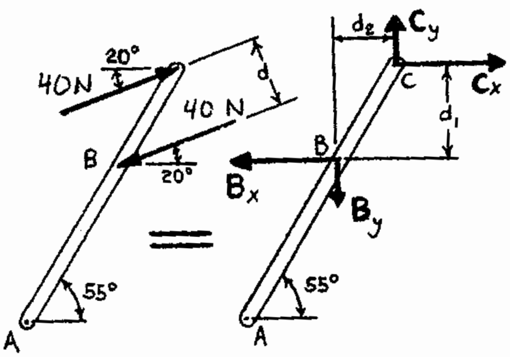
$485 \text{ lb}\cdot\text{in.} = [(6+d)\text{ in.}](35 \text{ lb}) + [(8+d)\text{ in.}](25 \text{ lb})$ $d = 1.250$ in. ◀



PROBLEM 3.74

Two parallel 40-N forces are applied to a lever as shown. Determine the moment of the couple formed by the two forces (*a*) by resolving each force into horizontal and vertical components and adding the moments of the two resulting couples, (*b*) by using the perpendicular distance between the two forces, (*c*) by summing the moments of the two forces about Point *A*.

SOLUTION



(*a*) We have $\Sigma \mathbf{M}_B: \quad -d_1 C_x + d_2 C_y = \mathbf{M}$

where $d_1 = (0.270 \text{ m})\sin 55°$

$= 0.22117 \text{ m}$

$d_2 = (0.270 \text{ m})\cos 55°$

$= 0.154866 \text{ m}$

$C_x = (40 \text{ N})\cos 20°$

$= 37.588 \text{ N}$

$C_y = (40 \text{ N})\sin 20°$

$= 13.6808 \text{ N}$

$\mathbf{M} = -(0.22117 \text{ m})(37.588 \text{ N})\mathbf{k} + (0.154866 \text{ m})(13.6808 \text{ N})\mathbf{k}$

$= -(6.1946 \text{ N}\cdot\text{m})\mathbf{k}$ or $\mathbf{M} = 6.19 \text{ N}\cdot\text{m}$ ↻ ◀

(*b*) We have $\mathbf{M} = Fd(-\mathbf{k})$

$= 40 \text{ N}[(0.270 \text{ m})\sin(55° - 20°)](-\mathbf{k})$

$= -(6.1946 \text{ N}\cdot\text{m})\mathbf{k}$ or $\mathbf{M} = 6.19 \text{ N}\cdot\text{m}$ ↻ ◀

(*c*) We have $\Sigma \mathbf{M}_A: \quad \Sigma(\mathbf{r}_A \times \mathbf{F}) = \mathbf{r}_{B/A} \times \mathbf{F}_B + \mathbf{r}_{C/A} \times \mathbf{F}_C = \mathbf{M}$

$$M = (0.390 \text{ m})(40 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55° & \sin 55° & 0 \\ -\cos 20° & -\sin 20° & 0 \end{vmatrix}$$

$$+ (0.660 \text{ m})(40 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55° & \sin 55° & 0 \\ \cos 20° & \sin 20° & 0 \end{vmatrix}$$

$= (8.9478 \text{ N}\cdot\text{m} - 15.1424 \text{ N}\cdot\text{m})\mathbf{k}$

$= -(6.1946 \text{ N}\cdot\text{m})\mathbf{k}$ or $\mathbf{M} = 6.19 \text{ N}\cdot\text{m}$ ↻ ◀