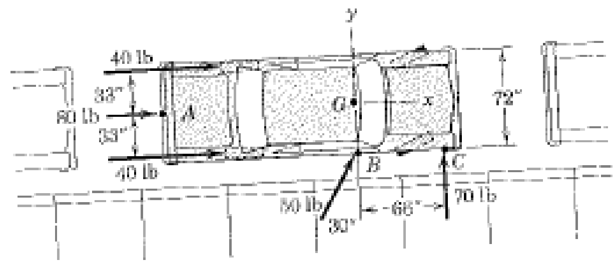
Engineering Statics – MECH 223

Review Problems for the Final Exam – Set 2

1. A rear-wheel-drive car is stuck in the snow between other parked cars as shown. In an attempt to free the car, three students exert forces on the car at points *A*,*B* and *C* while the driver's actions result in a forward thrust of 40 lb acting parallel to the plane of rotation of each rear wheel. Treating the problem as two-dimensional, determine the equivalent force-couple system at the car center of mass *G*, and locate position *x* of the point on the car centerline through which the resultant passes. Neglect all forces not shown.



Solution:

Use $\overset{\uparrow y}{\underset{}{\llcorner}}\!\!\rightarrow x$ $\circlearrowleft +$ system at G:

$$\underline{R} = \Sigma \underline{F} = (80 + 40 + 40 + 50 \sin 30°)\,\underline{i} + (50 \cos 30° + 70)\,\underline{j}$$

$$= \underline{185\,\underline{i} + 113.3\,\underline{j} \text{ lb}}$$

$$M_G = 70(66) + 50 \sin 30°\,(36) = 5520 \text{ lb-in.}$$

$$= \underline{460 \text{ lb-ft}}\ (\circlearrowleft)$$

For line of action of resultant:

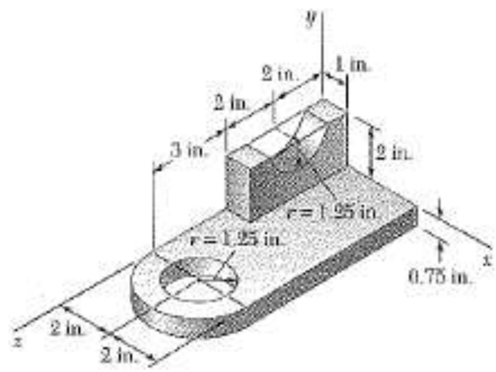
$$\underline{r} \times \underline{R} = \underline{M}_G$$

$$(x\,\underline{i} + y\,\underline{j}) \times (185\,\underline{i} + 113.3\,\underline{j}) = 460\,\underline{k}$$

$$113.3x - 185y = 460$$

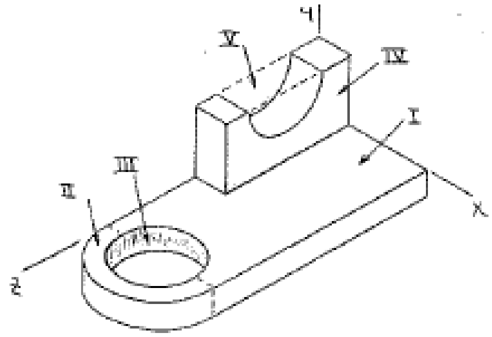
$$\underline{x = 4.06 \text{ ft}} \text{ when } y = 0.$$

2. For the machine element shown, locate the *z* coordinate of the center of gravity.



Solution:

FIRST ASSUME THAT THE MACHINE ELEMENT IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING VOLUME.

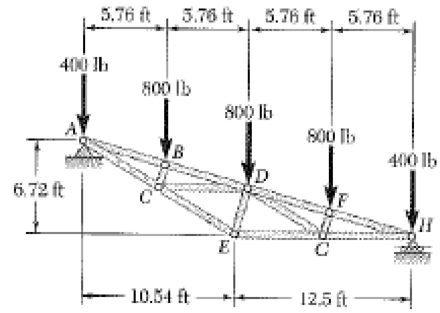


	V, IN^3	$\bar{z}$, IN.	$\bar{z}V$, IN^4
I	$(4)(0.75)(7) = 21$	3.5	73.5
II	$\frac{\pi}{2}(2)^2(0.75) = 4.7124$	$7 + \frac{4 \times 2}{3\pi} = 7.8488$	36.987
III	$-\pi(1.25)^2(0.75) = -3.6816$	7	-25.771
IV	$(1)(2)(4) = 8$	2	16
V	$-\frac{\pi}{2}(1.25)^2(1) = -2.4544$	2	-4.9088
Σ	27.576		95.807

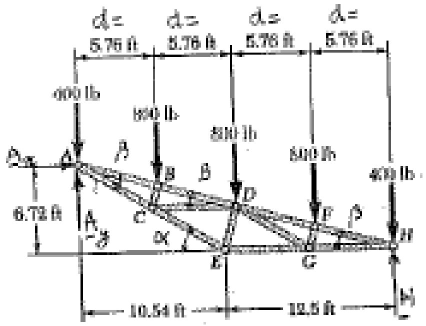
HAVE... $\bar{Z}\Sigma V = \Sigma \bar{z} V$: $\bar{Z}(27.576 \text{ IN}^3) = 95.807 \text{ IN}^4$

OR $\bar{Z} = 3.47$ IN.

3. Determine the force in each of the embers located to the right of *DE* for the inverted Howe roof truss shown. State whether each member is in tension or compression.



Solution:



FREE BODY: TRUSS

$+\circlearrowleft \Sigma M_A = 0:$

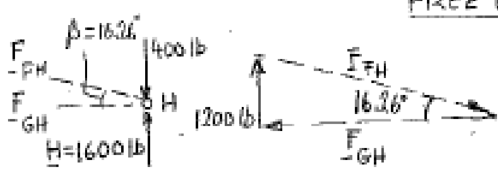
$H(4d) - (800\text{ lb})d - (800\text{ lb})(2d) - (800\text{ lb})(3d) - (400\text{ lb})(4d) = 0$

$H = 1600\text{ lb}\uparrow$

ANGLES: $\tan\alpha = \frac{6.72}{10.54}$ $\quad \alpha = 32.52^\circ$

$\tan\beta = \frac{6.72}{23.04}$ $\quad \beta = 16.26^\circ$

FREE BODY: JOINT H



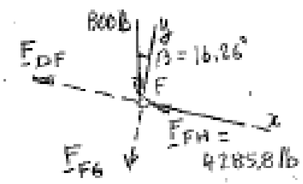
$F_{GH} = (1200\text{ lb})\cot 16.26$

$F_{GH} = 4114.3\text{ lb T}$

$F_{GH} = 4110\text{ lb T}$

$F_{FH} = \frac{1200\text{ lb}}{\sin 16.26^\circ} = 4285.8\text{ lb}$ $\quad F_{FH} = 4290\text{ lb C}$

FREE BODY: JOINT F



$+\nearrow \Sigma F_y = 0:$

$-F_{FG} - (800\text{ lb})\cos 16.26^\circ = 0$

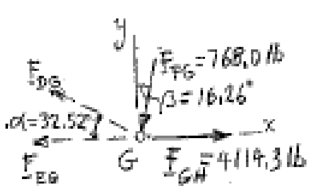
$F_{FG} = -768.0\text{ lb}$ $\quad F_{FG} = 768\text{ lb C}$

$\underrightarrow{+} \Sigma F_x = 0:$

$-F_{DF} - 4285.8\text{ lb} + (800\text{ lb})\sin 16.26^\circ = 0$

$F_{DF} = -4061.8$ $\quad F_{DF} = 4060\text{ lb C}$

FREE BODY: JOINT G



$+\uparrow \Sigma F_y = 0:$

$F_{DG}\sin 32.52^\circ - (768.0\text{ lb})\cos 16.26^\circ = 0$

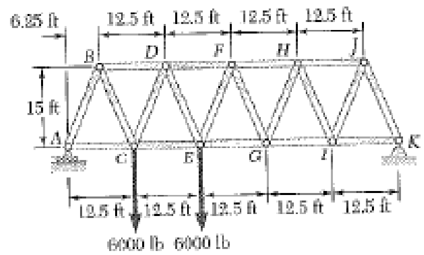
$F_{DG} = +1371.4\text{ lb}$

$F_{DG} = 1371\text{ lb T}$

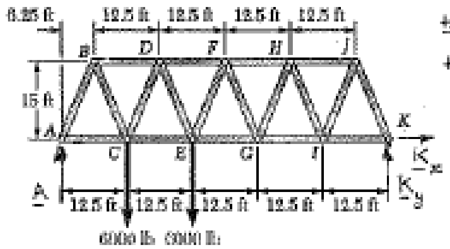
$\underrightarrow{+} \Sigma F_x = 0: \quad -F_{EG} + 4114.3\text{ lb} - (768.0\text{ lb})\sin 16.26^\circ - (1371.4\text{ lb})\cos 32.52^\circ = 0$

$F_{EG} = +2742.9\text{ lb}$ $\quad F_{EG} = 2740\text{ lb T}$

4. A Warren bridge truss is loaded as shown. Determine the force in members *EG*, *FG*, and *FH*.



Solution:



FREE BODY: TRUSS

$\xrightarrow{+}\Sigma F_x = 0$: $K_x = 0$

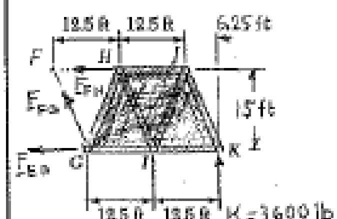
$+\circlearrowleft \Sigma M_A = 0$: $K_y(62.5\text{ ft}) - (6000\text{ lb})(12.5\text{ ft}) - (6000\text{ lb})(25\text{ ft}) = 0$

$K = K_y = 3600\text{ lb}\uparrow$

$+\uparrow\Sigma F_y = 0$: $A + 3600\text{ lb} - 6000\text{ lb} - 6000\text{ lb} = 0$

$A = 8400\text{ lb}\uparrow$

WE PASS A SECTION THROUGH MEMBERS EG, FG, AND FH, AND USE THE FREE BODY SHOWN.



$+\circlearrowleft\Sigma M_F = 0$:

$(3600\text{ lb})(31.25\text{ ft}) - F_{EG}(15\text{ ft}) = 0$

$F_{EG} = +7500\text{ lb}$, $F_{EG} = 7500\text{ lb}\ T$

$+\uparrow\Sigma F_y = 0$: $\frac{15}{16.25}F_{FG} + 3600\text{ lb} = 0$

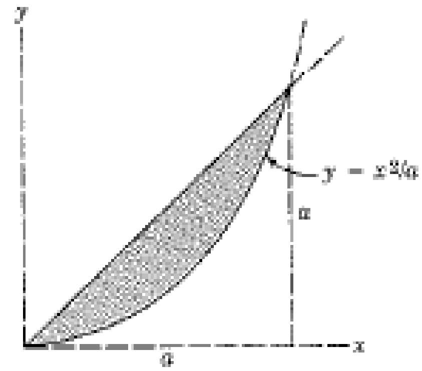
$F_{FG} = -3900\text{ lb}$, $F_{FG} = 3900\text{ lb}\ C$

$+\circlearrowleft\Sigma M_G = 0$:

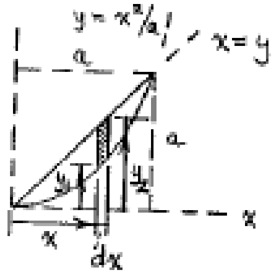
$F_{FH}(15\text{ ft}) + (3600\text{ lb})(25\text{ ft}) = 0$

$F_{FH} = -6000\text{ lb}$, $F_{FH} = 6000\text{ lb}\ C$

5. Determine the moments of inertia of the shaded area about the *x*- and *y*-axes. Use the same differential element for both calculations.



Solutions:



$$dA = (y_2 - y_1)\,dx = \left(x - \frac{x^2}{a}\right)dx$$

$$I_x = \int \frac{1}{3} y_2^3\,dx - \int \frac{1}{3} y_1^3\,dx = \frac{1}{3}\int_0^a \left(x^3 - \frac{x^6}{a^3}\right)dx$$

$$= \frac{1}{3}\left[\frac{x^4}{4} - \frac{x^7}{7a^3}\right]_0^a = \frac{a^4}{3}\left(\frac{1}{4} - \frac{1}{7}\right) = \underline{a^4/28}$$

$$I_y = \int x^2\,dA = \int x^2\left(x - \frac{x^2}{a}\right)dx = \int_0^a \left(x^3 - \frac{x^4}{a}\right)dx$$

$$= \left[\frac{x^4}{4} - \frac{x^5}{5a}\right]_0^a = \frac{a^4}{4} - \frac{a^4}{5} = \underline{a^4/20}$$