Curved Girder Deformation Prediction Effectiveness Using First-Order, Linear Geometric Finite-Element Models

D. L. Nevling[1] and D. G. Linzell, F.ASCE[2]

Abstract: This study examines whether a first-order, linear geometric finite-element static model analysis is capable of accurately predicting the deflection and rotation response of a curved steel I-girder bridge and curved steel I-girders. Girder deflections obtained during the erection of a horizontally-curved steel I-girder bridge and deflection and rotation data from curved-beam laboratory tests were used to determine if a linear geometric finite-element analysis could accurately predict their static behavior. Results indicate that the first-order, linear geometric finite-element model accurately predicts the deflection behavior of a horizontally-curved steel I-girder bridge and a curved steel I-girder beam tested under point loads. The results also indicate that the first-order, linear geometric finite-element model predicted the general rotation trends of the experimental beam accurately. However, the model overpredicted peak flange-tip displacements when compared with the experimental results. **DOI:** 10.1061/(ASCE)SC.1943-5576.0000076. *© 2011 American Society of Civil Engineers.*

CE Database subject headings: Bridges, girder; Curved beams; Geometry; Finite element method; Deformation.

Author keywords: Bridges; Curved beams; Linear geometric; Finite-element models.

Introduction

The complex geometry of horizontally-curved steel I-girders can lead to difficulties in determining the deflection and rotation responses of the curved girders. The curvature of the girders causes warping stresses to develop in the top and bottom flanges. The warping stresses and the vertical bending stresses result in higher stresses acting on curved beams when compared with straight beams. DeSantiago et al. (2005) concluded that curved bridges experience approximately 23.5% higher bending moments than straight bridges for an angle of curvature of 30°. Finite-element models are often created to aid in determining the behavior of curved steel I-girders. This study examines whether a first-order, linear geometric finite-element static model analysis is capable of accurately predicting the deflection and rotation response of a curved steel I-girder bridge and curved steel I-girders. Past studies indicate that to accurately predict the deformation response of curved steel I-girders, a second-order, nonlinear geometric finite-element analysis is needed. However, second-order, nonlinear geometric finite-element analyses are complex models and require additional computational run time when compared with first-order, linear geometric finite-element models. Recognizing the level of accuracy that first-order, linear geometric models provide when attempting to predict the static deformation response of curved steel I-girders and steel curved I-girder systems would be beneficial.

Background

A number of recent studies have concluded that to accurately model the behavior of horizontally-curved, steel I-girders, either by themselves or in a curved bridge system, a large-displacement, geometrically nonlinear analysis needs to be utilized. Shanmugam et al. (1995) conducted a series of load tests on beams with various values of horizontal radius of curvature to determine their ultimate load behavior. The results from the tests were compared with results from a three-dimensional finite-element model created in Abaqus (2003) that included both material and large-displacement geometric nonlinear effects. It was concluded that the Abaqus finite-element analysis was capable of accurately predicting the ultimate load capacity of the experimentally tested horizontally-curved steel beams. First-order, linear geometric analysis results were not reported.

Pi et al. (2000) developed a three-dimensional finite-element model for the nonlinear inelastic analysis of I-beams curved in plan and concluded that the model was accurate, efficient, and economical based on comparisons with two independent sets of test data. Pi et al. (2001) also determined that the need for using a large rotation model may depend on the magnitude of the radius of curvature of the girder. First-order, linear geometric analysis results were not reported.

Bradford et al. (2001) used a nonlinear inelastic curved beam finite-element model to investigate the behavior of horizontally-curved steel I-girder beams under construction loading, concluding that a nonlinear analysis is needed to correctly predict deflections of composite curved girders. The finite-element model used for this study was validated through comparisons with laboratory test results for curved beams. A similar finite-element model was used to conduct a parametric study of the effects of included angle, slenderness, residual stresses, and lateral bracing on composite, curved I-girder structural behavior. The conclusion that geometrically nonlinear analysis was needed was based on parametric study results and curved beam theory. Again, first-order, linear geometric analysis results were not reported.

Earls and Chavel (2001) utilized a geometric nonlinear three-dimensional finite-element model to investigate the behavior of

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[1] Civil Associate, Michael Baker Jr., Inc., 272 Bendix Rd., Suite 400, Virginia Beach, VA 23454 (corresponding author). E-mail: dnevling@mbakercorp.com

[2] Associate Professor and Director, Dept. of Civil and Environmental Engineering, Protective Technology Center, The Pennsylvania State University, 231L Sackett Building, University Park, PA 16802. E-mail: dlinzell@engr.psu.edu

Note. This manuscript was submitted on November 18, 2009; approved on July 22, 2010; published online on April 15, 2011. Discussion period open until October 1, 2011; separate discussions must be submitted for individual papers. This paper is part of the ***Practice Periodical on Structural Design and Construction***, Vol. 16, No. 2, May 1, 2011. ©ASCE, ISSN 1084-0680/2011/2-56–61/$25.00.

Table 1. Elements for Structure 207 Finite-Element Model

Bridge component	Element type	Abaqus element designation	Degrees of freedom	Notes
Web	Shell	S4R	6	
Flanges	Beam	B31	6	Offset not included
Cross frames	Beam	B31OS	6	Top and bottom chords and diagonals modeled individually
Transverse stiffeners	Beam	B31	6	
Slab	Shell	S4R	6	Connected to top flanges using rigid links.[a]

[a] The rigid links had a very minimal mass and an extremely large stiffness value (2.5 times the elastic modulus of steel).

a horizontally-curved steel I-girder bridge during construction. Shura (2005) also included geometrically nonlinear effects in a grillage model of a horizontally-curved steel I-girder during construction. Neither Earls and Chavel (2001) nor Shura (2005) presented results from a linear model.

All studies concluded that a geometrically nonlinear analysis is needed for curved I-girders and curved I-girder bridge systems, but none reported results from a first-order, linear geometric static analysis. As a result of these findings, the current study compares deflections and rotations of a moderately curved steel I-girder bridge and a severely curved steel beam to linear geometric finite-element analysis results to determine if this level of analysis accurately predicts the curved bridge and beam behavior under static loads.

Bridge Finite-Element Model

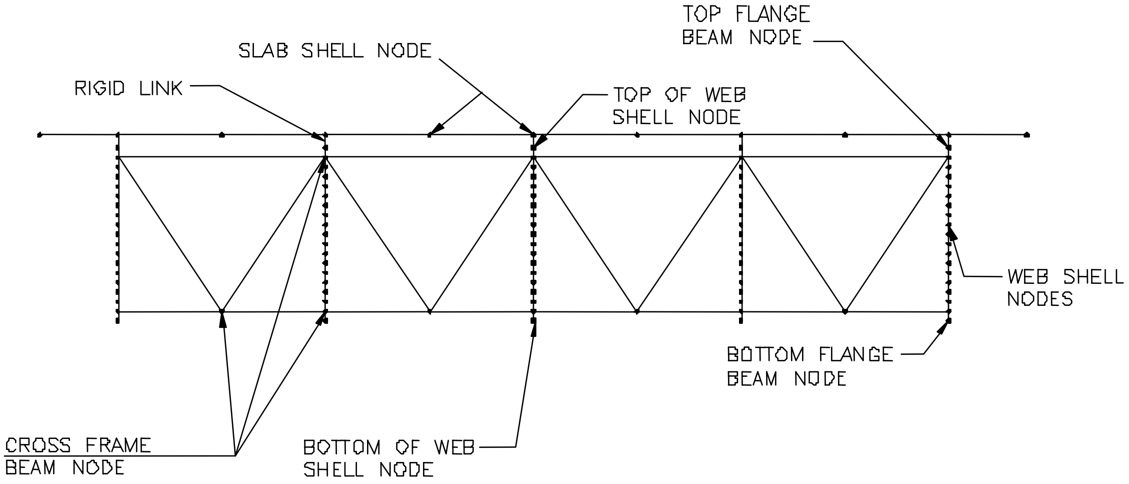
All finite-element models for this study were created with Abaqus/Standard (2003). For the examined bridge (Structure 207), girder webs were modeled with 18 ABAQUS S4R shell elements through their depth, which resulted in element-edge lengths of 152 mm (6 in.). Nodes along the girder length were placed at approximately 152 mm (6 in.) to keep the element aspect ratio as close to 1 as possible. A description of Structure 207 is provided in the next section, Table 1 lists the element types used to model each component of the structure, and Fig. 1 shows the details of the three-dimensional finite-element model created in Abaqus.

All components of the model were assigned corresponding nominal material properties for structural steel and concrete. The assumption was made that all parts of the model would remain in the elastic range because the structure is analyzed during construction with no live load present. Girder support conditions matched actual bearing designs with both abutments for Girders 1 and 5 being modeled as nonguided expansion pot bearings having deflection restrained in the vertical direction only. Supports at the abutments for Girders 2, 3, and 4 were modeled by using tangentially-guided pot bearings. Deflection was restrained in the y-direction and the z-direction (see Fig. 2). All pier bearings were modeled as transversely-guided pot bearings with deflection restrained in the x-direction and the z-direction.

Bridge Description and Instrumentation

Structure 207 is a two-span continuous bridge composed of five horizontally-curved steel plate girders located in Centre County, Pennsylvania. The centerline radius of curvature is 585 m (1,921 ft.). Fig. 2 shows a plan view of the superstructure and Fig. 3 shows a typical cross section of the bridge. More detailed information on Structure 207 is available in Shura (2005) and Nevling (2008). Structure 207 was selected for the current study because deflection data are available for all five girders during construction.

Measured girder deflection values were obtained directly from Greenman-Pederson, Inc. (GPI), who conducted a scanning study of Structure 207 with Cyrax Laser Technology (Shura 2005). Girder deflection values were obtained during two stages of construction: (1) immediately after placement of the entire steel superstructure; and (2) after placement of the entire concrete deck. Deflection readings were taken at 24 locations along all the girders.



NOTE: STIFFENER BEAM NODES CORRESPOND TO WEB SHELL NODE LOCATIONS

Fig. 1. Detailed model geometry for typical bridge cross section

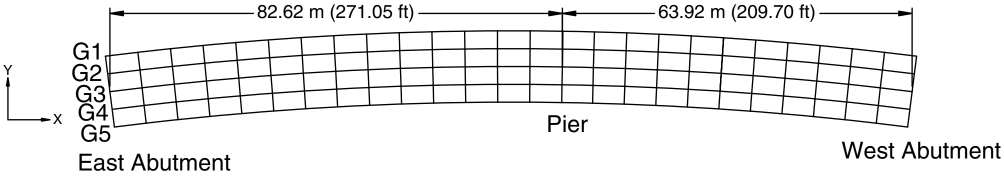


Fig. 2. Structure 207 plan view; the z-axis is parallel to the girder webs

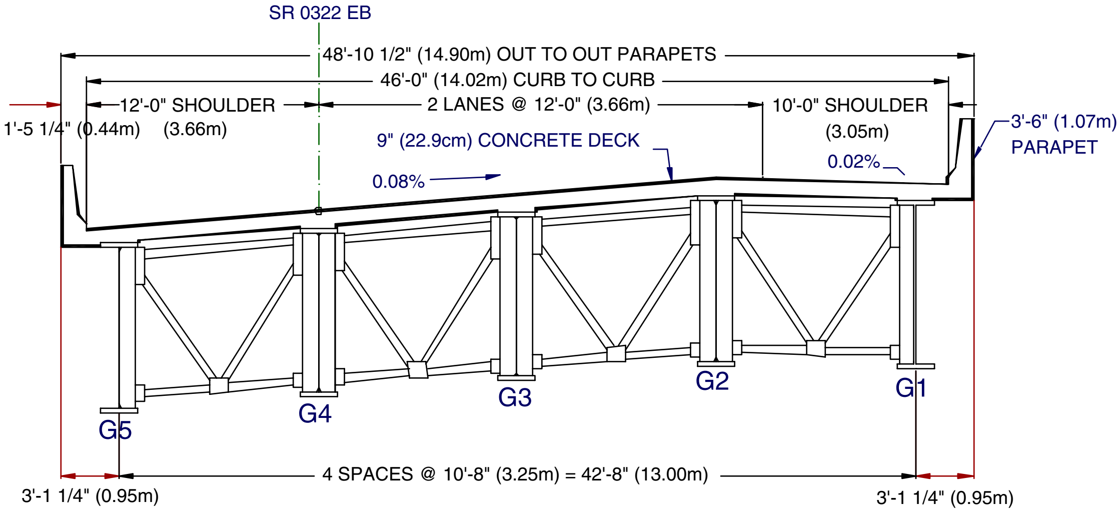


Fig. 3. Typical cross section of structure 207 (Shura 2005, with permission)

These results are presented to provide another level of comparison for the linear geometric three-dimensional finite-element model.

Horizontally-Curved Steel I-Girder Bridge Comparisons

This section contains deflection comparisons between the Abaqus three-dimensional finite-element model and the field data for a large-radius, long-span, plate-girder bridge. Vertical deflection data obtained from the field test included changes in deformations for all girders between erection of the entire steel superstructure and placement of the deck as obtained by Greenman-Pederson, Inc. Figs. 4 and 5 show the deflection for Girders 1 and 5, respectively, which are representative of the deflections for the remaining girders. Figs. 4 and 5 show that the first-order, linear geometric Abaqus three-dimensional finite-element static model predicts the general behavior of girder vertical deflections observed in the field and also accurately predicts the maximum deflection of the girders observed in Spans 1 and 2. The deflections in Figs. 4 and 5 represent the change in deflection between the following construction stages: (1) entire steel superstructure is in place; and (2) deck is in place and has achieved its 7-day strength. Accurate deflection control during the placement of the concrete deck on the horizontally-curved steel I-girder is necessary to ensure that unwanted and/or excessive movement of the girders does not occur. Figs. 4 and 5 also contain the percentage-difference comparisons between the girder deflections observed in the field and those obtained from the first-order linear geometric Abaqus three-dimensional finite-element model. The percentage-difference values for the Abaqus first-order linear geometric finite-element model vary from 3.2 to 10.6%.

After analyzing the displacement results for the Abaqus three-dimensional first-order, linear geometric finite-element model and the field tests, the Abaqus three-dimensional finite-element model was deemed capable of accurately predicting the vertical deflection behavior of Structure 207 during construction. Structure 207 has a large R/L ratio and therefore, the next section compares Abaqus three-dimensional first-order, linear geometric finite-element model results to experimental results for a curved girder with a small R/L ratio.

Comparisons of Curved Beam Results

A first-order, linear geometric three-dimensional finite-element model similar to the one used for the large-radius curved bridge in the previous section was utilized to predict deformations obtained for a curved girder with a very small R/L ratio tested under static loads. To determine if the first-order, linear geometric finite-element modeling technique is valid for a broad range of curvatures, finite-element model deformations and rotations were compared with the laboratory test results.

Heins and Spates (1968) conducted an experimental study on a single horizontally-curved steel girder (Type 7I15.3) to validate a computer program that calculated internal forces and external deflections and rotations of single horizontally-curved I-girders. The web had a depth of 177.8 mm (7.0 in.) and a thickness of 6.35 mm (0.25 in.). The flanges were 93.0 by 10.0 mm (3.7 in. by 0.4 in.). The girder was 9.1-m (30.0-ft.) long and had a radius of curvature of 15.2 m (50.0 ft.). The ends of the girder were encased in concrete blocks to replicate fixed-end conditions, resulting in a clear span of 8.2 m (27.0 ft.). The R/L ratio of the girder was 1.85, indicating an extreme horizontal curvature. The girder was tested under concentrated vertical loads at (1) midspan, and (2) the three-tenths point.

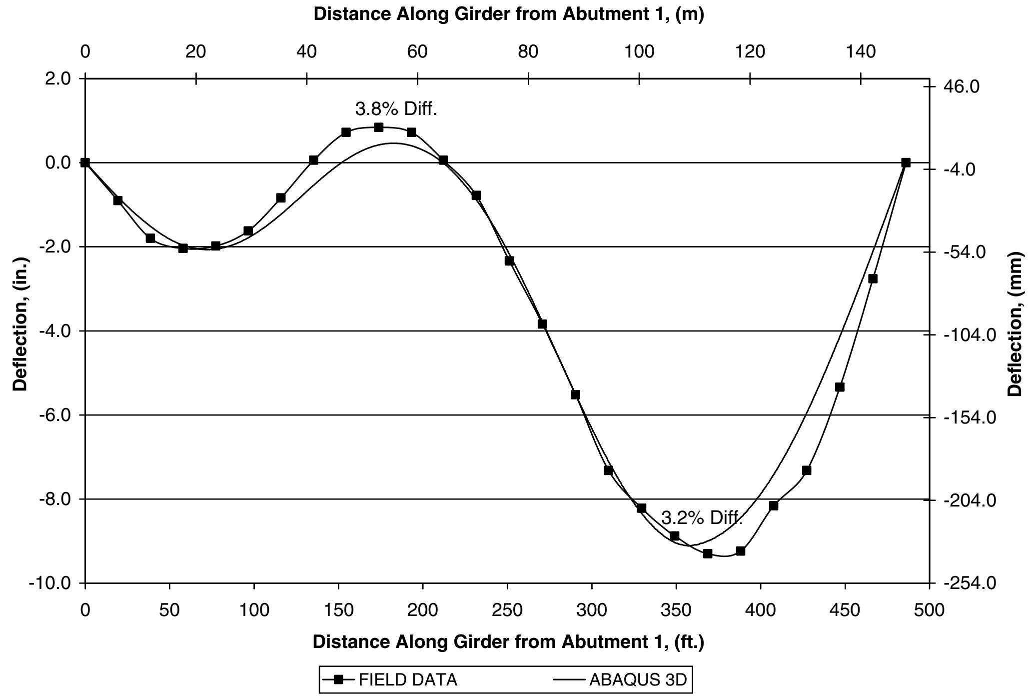


Fig. 4. Change in Girder 1 vertical displacements between erection of entire steel superstructure and placement of slab

The maximum applied load in each case was 4,448 N (1,000 lbs). The deflections were recorded at the tenths points along the unrestrained length of the girders. The rotations were recorded at only 4 locations along the girder [0.5, 1.7, 2.9, and 4.6 m (1.5, 5.6, 9.6, and 15.0 ft.)].

A three-dimensional finite-element model of the Heins and Spates test beam was created in Abaqus following the modeling techniques described previously. Seven S4R shell elements were used throughout the depth with nodes placed at approximately 25 mm (1 in.) intervals along the girder to keep element aspect ratios close to 1. The nodes along the ends of the beam were completely restrained to replicate the fixed support conditions. The beam was analyzed under the two previously discussed loading conditions. Results are presented for the concentrated load at midspan. Results for the concentrated load at the three-tenths point are similar to the results for the concentrated load at midspan.

Figs. 6 and 7 show deformation comparisons between the test and the current study's Abaqus models for the concentrated load placed at midspan. Fig. 6 shows that the first-order, linear geometric Abaqus model predicted the experimental vertical deflection

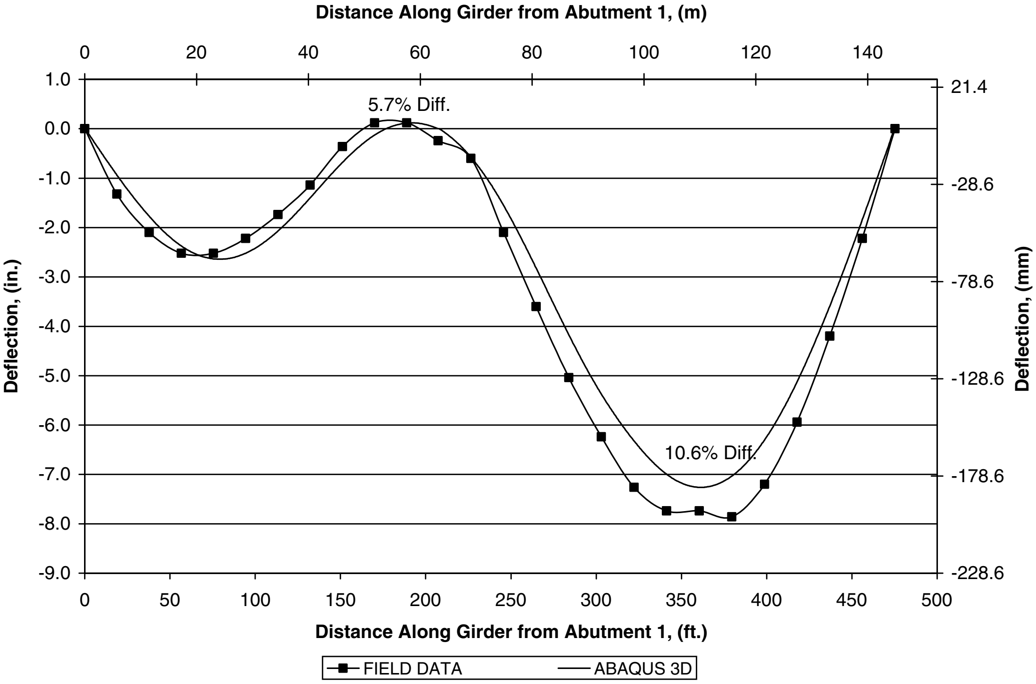


Fig. 5. Change in Girder 5 vertical displacements between erection of entire steel superstructure and placement of slab

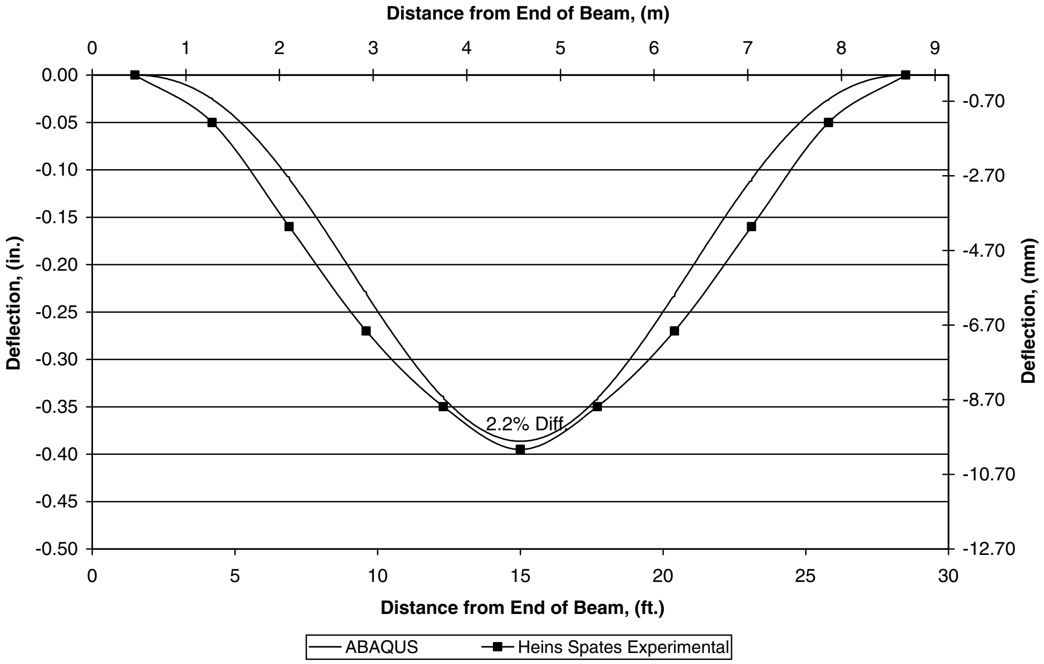


Fig. 6. Vertical deflection results comparison for the concentrated load at midspan

behavior accurately. The percentage difference between the experimental and Abaqus model deflection is 2.2% at the midspan of the beam. Fig. 7 shows that the Abaqus model predicted the general rotation behavior of the experimental beam accurately. However the Abaqus model overpredicted peak flange-tip displacements when compared with the experimental results. The experimental beam was only instrumented at four locations with rotation gauges compared with 11 deflection gauges. The percentage difference values between the experimental and the model flange-tip displacement results vary widely, between −13.8 and −364.7%. However, the maximum magnitudes observed from the laboratory tests were quite small, between 0.141 and 1.645 mm (0.006 and 0.065 in.), and were less than the least conservative tolerances given in the American Welding Society (AWS) Structural Welding Code (2004), which limit the combined warpage and tilt of the flange to 1% of the flange width [0.93 or 6 mm (0.037 or 0.25 in.)]. Possible explanations for these differences are: (1) the flange-tip deflection values were less than the least conservative limits specified in the AWS Structural Welding Code (2004), and therefore, could be considered negligible and difficult for finite-element model to predict; (2) encasing the ends of the experimental beams in concrete blocks restrained the rotation more than could be

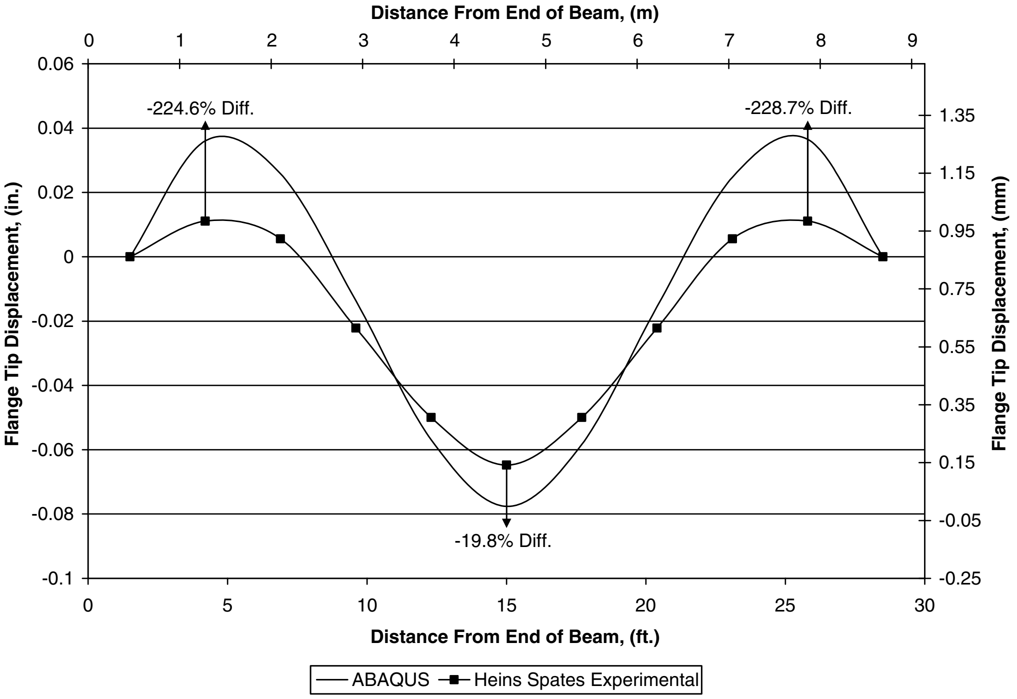


Fig. 7. Flange-tip displacement results comparison for the concentrated load at midspan

accounted for in the model. However, the rotations caused very minimal vertical deflection of the flange tips. Additional research is needed to establish accurate and efficient finite-element modeling techniques to predict rotations in curved beams.

Conclusions

Deformation results for first-order, linear geometric Abaqus models of an actual two-span continuous curved steel I-girder bridge and an experimental beam tested by Heins and Spates (1968) were presented and compared with field and experimental results, respectively, to investigate whether first-order, linear geometric finite-element model analysis can accurately predict the static behavior of curved I-girders and curved I-girder bridge systems.

The conclusions from this study are:

- The first-order, linear geometric three-dimensional finite-element model utilized in the current study accurately predicted the vertical deflection response of a curved (R/L of 4.00) steel I-girder bridge during construction;
- The model utilized for this study also accurately predicted the vertical deflection response of the Heins and Spates experimental beam (R/L of 1.85);
- The model did not accurately predict the rotation behavior of an experimental curved beam. The rotation magnitudes observed in the laboratory were very small, less than the AWS Bridge Welding Code's (2004) least conservative limits, which led to discrepancies between the finite-element model and laboratory results. As magnitudes of observed rotation increased, the accuracy of the finite-element model in predicting curved beam behavior increased; and
- Because of the Abaqus model's ability to accurately predict the vertical deflection behavior of the Heins and Spates experimental beam (R/L of 1.85) and the vertical deflection behavior of the curved bridge that was examined (R/L of 4.00), it was concluded that a first-order, linear geometric model is capable of predicting static vertical deflection behavior of horizontally-curved steel bridges. These results do not account for moving live loads or dynamic loads. Further investigation to definitively determine if first-order, linear geometric finite-element analysis can accurately predict the deformation response of curved steel I-girder bridge systems with varying R/L ratios is warranted. Additional research should also be completed to establish accurate and efficient finite-element modeling techniques for predicting rotations in curved beams.

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