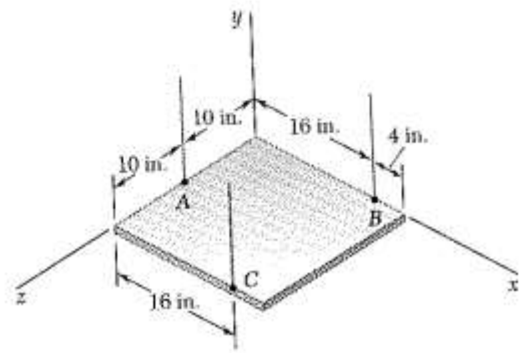
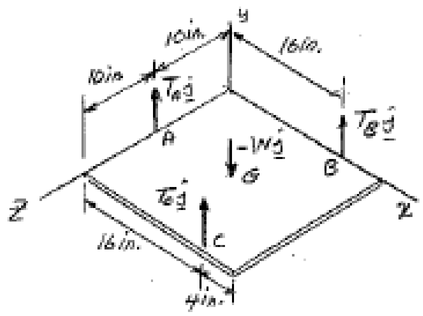
Example from midterm: The 20 X 20-in square plate weighs 56 lb and is supported by three vertical wires as shown in the figure to the right.



(a) Determine the tension in each wire.

Solution:



The center of mass for the application of *W* is at the center of the plate.

Equilibrium of moments around *z* axis passing through *B* and *C*

$$\sum M_{Bz} = 0 = W * 6\ in - T_A * 16\ in \ \Rightarrow\ T_A = \frac{56\ lb * 6\ in}{16\ in} = 21\ lb$$

Because of symmetry, $T_C = T_B = T$.

Equilibrium of forces in the *y* direction:

$$\sum F_y = 0 = 2T + T_A - W \quad \Rightarrow \quad T = \frac{56\ lb - 21\ lb}{2} = 17.5\ lb$$

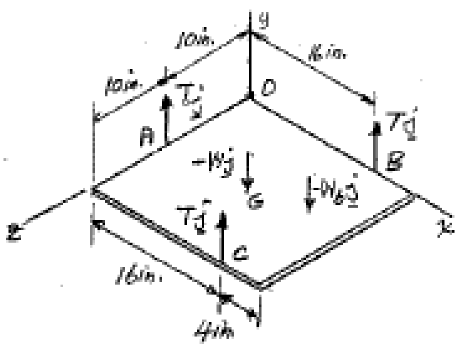
Answer:

$T_A = 21\ lb$

$T_B = T_C = 17.5\ lb$

(b) If a vertical force of 10 lb is applied down on the plate, what is the point of application if it results in all tensions being equal? What is the corresponding tension?

Solution:



Now, $W_b = 10\ lb$ is applied to the plate at a point (x_b, z_b).

Since all the tensions are equal, from an equilibrium of forces in the y direction:

$$\sum F_y = 0 = 3T - W - W_b \quad \Longrightarrow \quad T = \frac{56\ lb + 10\ lb}{3} = 22\ lb$$

From symmetry, to maintain an equilibrium of moments around the x axis that passes through A, and because $T_B = T_C$, it follows that $z_b = 10\ in$.

To calculate the x_b consider equilibrium of moments around the z axis passing through B.

$$\sum M_{Bz} = 0 = W_b * (16\ in - x_b) + W * 6\ in - T * 16\ in$$

$$(16\ in - x_b) = \frac{22\ lb * 16\ in - 56\ lb * 6\ in}{10\ lb} = 1.6\ in$$

$$x_b = 16\ in - 1.6\ in = 14.4\ in$$

(c) (5 points) What is the maximum distance (d) from the z axis for a 150 lb force down to be applied on the plate without the plate tipping?

Solution:

For a plate to tip means in this case to start rotating around the BC axis. The extreme case, just before tipping means that $T_A = 0$. From equilibrium of moments around BC gives us:

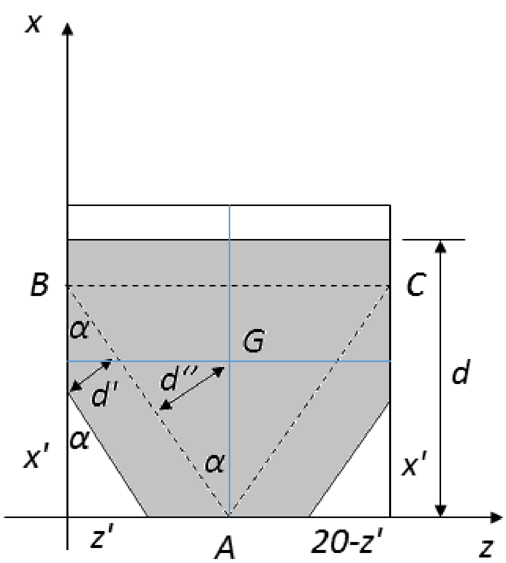
$$\sum M_{BC} = 0 = W * 6\ in - W_b * (d - 16) in \quad \Longrightarrow \quad d = \frac{(56\ lb * 6\ in)}{150\ lb} + 16\ in = 18.24\ in$$

(d) Bonus Show on a sketch the area of the plate over which the force in part (c) can act without the plate tipping. Mark relevant distances.

Solution:

The plate can tip around one of the three axes *BC*, *AB* and *AC*. In each of the extreme cases, just before tipping, the tension in the corresponding third cable is zero. In the previous part we calculate the distance *d* for the case of tipping around the *BC* axis.

The area that the force can be applied to without tipping the plate is marked in gray in the top view of the plate:



$$\tan\alpha = \frac{10}{16} \quad \Rightarrow \quad \alpha = 32°$$

$$d'' = 10\ in * \sin\alpha = 5.3\ in$$

$$\sum M_{AB} = 0 = W_b * d' - W * d''$$

$$d' = \frac{W * d''}{W_b} = \frac{56\ lb * 5.3\ in}{150\ lb} = 1.98\ in$$

$$(16 - x') = \frac{d'}{\sin\alpha} \quad \Rightarrow \quad x' = 16\ in - \frac{d'}{\sin\alpha} = 12.27\ in$$

$$z' = x' * \tan\alpha = 7.67\ in$$

$$20 - z' = 12.33\ in$$