Homework #11 Solution

PROBLEM 8.3

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 100$ lb.

SOLUTION

Assume equilibrium:

\[ \sum F_y = 0: \quad F + (45 \text{ lb}) \sin 30^\circ - (100 \text{ lb}) \cos 40^\circ = 0 \]

\[ F = +54.0 \text{ lb} \]

\[ \sum F_x = 0: \quad N - (45 \text{ lb}) \cos 30^\circ - (100 \text{ lb}) \sin 40^\circ = 0 \]

\[ N = 103.2 \text{ lb} \]

(a) Maximum friction force:

\[ F_m = \mu_s N \]

\[ = 0.40(103.2 \text{ lb}) \]

\[ = 41.30 \text{ lb} \]

We note that $F > F_m$. Thus, block moves up.

(b) Actual friction force:

\[ F = F_k = \mu_k N = 0.30(103.2 \text{ lb}) = 30.97 \text{ lb}, \]

\[ F = 31.0 \text{ lb} \\angle 30.0^\circ \]
PROBLEM 8.5

Determine the smallest value of $P$ required to (a) start the block up the incline, (b) keep it moving up, (c) prevent it from moving down.

SOLUTION

(a) To start block up the incline:

$\mu_e = 0.40$

$\phi_e = \tan^{-1} 0.40 = 21.80^\circ$

From force triangle:

\[
\frac{P}{\sin 51.80^\circ} = \frac{45 \text{ lb}}{\sin 28.20^\circ}
\]

$P = 74.8 \text{ lb} \uparrow$

(b) To keep block moving up:

$\mu_k = 0.30$

$\phi_k = \tan^{-1} 0.30 = 16.70^\circ$

From force triangle:

\[
\frac{P}{\sin 46.70^\circ} = \frac{45 \text{ lb}}{\sin 33.30^\circ}
\]

$P = 59.7 \text{ lb} \uparrow$

(c) To prevent block from moving down:

From force triangle:

\[
\frac{P}{\sin 8.20^\circ} = \frac{45 \text{ lb}}{\sin 71.80^\circ}
\]

$P = 6.76 \text{ lb} \uparrow$
**PROBLEM 8.7**

The 80-lb block is attached to link AB and rests on a moving belt. Knowing that $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the magnitude of the horizontal force $P$ that should be applied to the belt to maintain its motion (a) to the right, (b) to the left.

**SOLUTION**

We note that link AB is a two-force member, since there is motion between belt and block $\mu_k = 0.20$ and $\phi_k = \tan^{-1} 0.20 = 11.31^\circ$.

(a) Belt moves to right

**Free body: Block**

Force triangle:

$$\frac{R}{\sin 120^\circ} = \frac{80 \text{ lb}}{\sin 48.69^\circ}$$

$$R = 92.23 \text{ lb}$$

**Free body: Belt**

$$+ \Sigma F_x = 0: \quad P - (92.23 \text{ lb}) \sin 11.31^\circ$$

$$P = 18.089 \text{ lb}$$

(b) Belt moves to left

**Free body: Block**

Force triangle:

$$\frac{R}{\sin 60^\circ} = \frac{80 \text{ lb}}{\sin 108.69^\circ}$$

$$R = 73.139 \text{ lb}$$

**Free body: Belt**

$$+ \Sigma F_x = 0: \quad (73.139 \text{ lb}) \sin 11.31^\circ - P = 0$$

$$P = 14.344 \text{ lb}$$
PROBLEM 8.8

The coefficients of friction between the block and the rail are \( \mu_s = 0.30 \) and \( \mu_k = 0.25 \). Knowing that \( \theta = 65^\circ \), determine the smallest value of \( P \) required (a) to start the block moving up the rail, (b) to keep it from moving down.

SOLUTION

(a) To start block up the rail:

\[ \mu_s = 0.30 \]
\[ \phi = \tan^{-1} 0.30 = 16.70^\circ \]

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500 N

\[ P \]
\[ \sin 51.70^\circ = \frac{500 \text{ N}}{\sin (180^\circ - 25^\circ - 51.70^\circ)} \]

\( P = 403 \text{ N} \)
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(b) To prevent block from moving down:

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500 N

\[ P \]
\[ \sin 18.30^\circ = \frac{500 \text{ N}}{\sin (180^\circ - 25^\circ - 18.30^\circ)} \]

\( P = 229 \text{ N} \)
PROBLEM 8.13

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force $P$ required to start the 30-kg block moving if cable $AB$ (a) is attached as shown, (b) is removed.

SOLUTION

(a) Free body: 20-kg block

\[ W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N} \]
\[ F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N} \]

\[ +\text{ } \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N} \]

Free body: 30-kg block

\[ W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N} \]
\[ N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N} \]
\[ F_2 = \mu_k N_2 = 0.3(490.5 \text{ N}) = 147.15 \text{ N} \]

\[ -\text{ } \Sigma F = 0: \quad P - F_1 - F_2 - T = 0 \]
\[ P = 78.48 \text{ N} + 196.2 \text{ N} + 78.48 \text{ N} = 353.2 \text{ N} \]

(b) Free body: Both blocks

Blocks move together

\[ W = (50 \text{ kg})(9.81 \text{ m/s}^2) = 490.5 \text{ N} \]

\[ +\text{ } \Sigma F = 0: \quad P - F = 0 \]
\[ P = \mu_k N = 0.3(490.5 \text{ N}) = 147.15 \text{ N} \]

\[ P = 196.2 \text{ N} \]
PROBLEM 8.23

A slender rod of length $L$ is lodged between peg $C$ and the vertical wall and supports a load $P$ at end $A$. Knowing that the coefficient of static friction between the peg and the rod is 0.15 and neglecting friction at the roller, determine the range of values of the ratio $L/a$ for which equilibrium is maintained.

SOLUTION

FBD rod:

Free-body diagram: For motion of $B$ impending upward:

$$\sum M_B = 0: \quad PL \sin \theta - N_C \left( \frac{a}{\sin \theta} \right) = 0$$

$$N_C = \frac{PL}{a \sin^2 \theta} \quad (1)$$

$$\sum F_y = 0: \quad N_C \sin \theta - \mu_s N_C \cos \theta - P = 0$$

$$N_C (\sin \theta - \mu \cos \theta) = P$$

Substitute for $N_C$ from Eq. (1), and solve for $a/L$.

$$\frac{a}{L} = \sin^2 \theta (\sin \theta - \mu_s \cos \theta) \quad (2)$$

For $\theta = 30^\circ$ and $\mu_s = 0.15$:

$$\frac{a}{L} = \sin^2 30^\circ (\sin 30^\circ - 0.15 \cos 30^\circ)$$

$$\frac{a}{L} = 0.09252 \quad \frac{L}{a} = 10.808$$

For motion of $B$ impending downward, reverse sense of friction force $F_C$. To do this we make $\mu_s = -0.15$ in Eq. (2).

Eq. (2):

$$\frac{a}{L} = \sin^2 30^\circ (\sin 30^\circ - (-0.15) \cos 30^\circ)$$

$$\frac{a}{L} = 0.15748 \quad \frac{L}{a} = 6.350$$

Range of values of $L/a$ for equilibrium:

$$6.35 \leq \frac{L}{a} \leq 10.81$$
PROBLEM 8.26

A 6.5-m ladder $AB$ leans against a wall as shown. Assuming that the coefficient of static friction $\mu_s$ is the same at $A$ and $B$, determine the smallest value of $\mu_s$ for which equilibrium is maintained.

SOLUTION

Free body: Ladder

Motion impending:

\[ F_A = \mu_s N_A \]
\[ F_B = \mu_s N_B \]

\[ \Sigma M_A = 0: \quad W(1.25 \text{ m}) - N_B(6 \text{ m}) - \mu_s N_B(2.5 \text{ m}) = 0 \]

\[ N_B = \frac{1.25W}{6 + 2.5\mu_s} \quad (1) \]

\[ \Sigma F_y = 0: \quad N_A + \mu_s N_B - W = 0 \]

\[ N_A = W - \mu_s N_B \]

\[ N_A = W - \frac{1.25\mu_s W}{6 + 2.5\mu_s} \quad (2) \]

\[ \Sigma F_x = 0: \quad \mu_s N_A - N_B = 0 \]

Substitute for $N_A$ and $N_B$ from Eqs. (1) and (2):

\[ \mu_s W - \frac{1.25\mu_s^3 W}{6 + 2.5\mu_s} = \frac{1.25W}{6 + 2.5\mu_s} \]

\[ 6\mu_s + 2.5\mu_s^3 - 1.25\mu_s^2 = 1.25 \]

\[ 1.25\mu_s^2 + 6\mu_s - 1.25 = 0 \]

\[ \mu_s = 0.2 \]

and

\[ \mu_s = -5 \quad \text{(Discard)} \]

\[ \mu_s = 0.200 \]
PROBLEM 8.39

Knowing that the coefficient of static friction between the collar and the rod is 0.35, determine the range of values of \( P \) for which equilibrium is maintained when \( \theta = 50^\circ \) and \( M = 20 \text{ N m} \).

SOLUTION

Free body member \( AB \):

\( BC \) is a two-force member.

\[ \Sigma M_A = 0: \quad 20 \text{ N m} - F_{BC} \cos 50^\circ (0.1 \text{ m}) = 0 \]

\[ F_{BC} = 311.145 \text{ N} \]

Motion of \( C \) impeding upward:

\[ \Sigma F_x = 0: \quad (311.145 \text{ N}) \cos 50^\circ - N = 0 \]

\[ N = 200 \text{ N} \]

\[ \Sigma F_y = 0: \quad (311.145 \text{ N}) \sin 50^\circ - P - (0.35)(200 \text{ N}) = 0 \]

\[ P = 168.351 \text{ N} \]

Motion of \( C \) impeding downward:

\[ \Sigma F_x = 0: \quad (311.145 \text{ N}) \cos 50^\circ - N = 0 \]

\[ N = 200 \text{ N} \]

\[ \Sigma F_y = 0: \quad (311.145 \text{ N}) \sin 50^\circ - P + (0.35)(200 \text{ N}) = 0 \]

\[ P = 308.35 \text{ N} \]

Range of \( P \):

\[ 168.4 \text{ N} \leq P \leq 308 \text{ N} \]