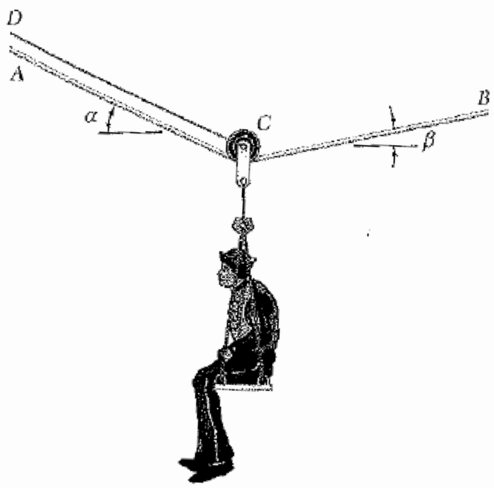
Homework #3 Solution

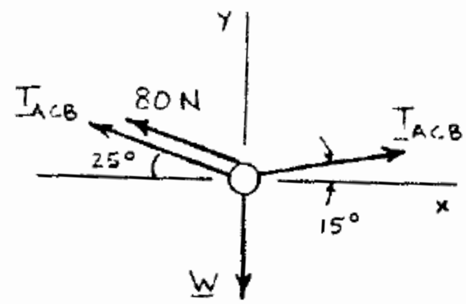


PROBLEM 2.54

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable ACB and is pulled at a constant speed by cable CD. Knowing that $\alpha = 25°$ and $\beta = 15°$ and that the tension in cable CD is 80 N, determine (*a*) the combined weight of the boatswain's chair and the sailor, (*b*) in tension in the support cable ACB.

SOLUTION

Free-Body Diagram



$\overset{+}{\rightarrow} \Sigma F_x = 0$: $T_{ACB} \cos 15° - T_{ACB} \cos 25° - (80\text{ N}) \cos 25° = 0$

$T_{ACB} = 1216.15\text{ N}$

$+\uparrow \Sigma F_y = 0$: $(1216.15\text{ N}) \sin 15° + (1216.15\text{ N}) \sin 25°$

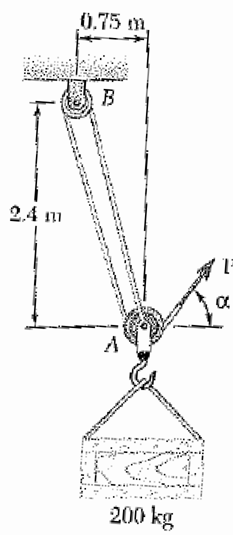
$+ (80\text{ N}) \sin 25° - W = 0$

$W = 862.54\text{ N}$

(*a*) $W = 863\text{ N}$ ◄

(*b*) $T_{ACB} = 1216\text{ N}$ ◄

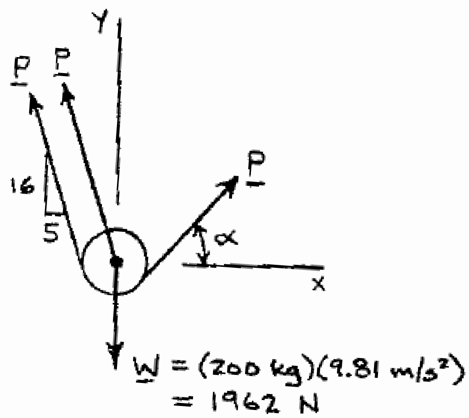
PROBLEM 2.66



A 200-kg crate is to be supported by the rope-and-pulley arrangement shown. Determine the magnitude and direction of the force **P** that must be exerted on the free end of the rope to maintain equilibrium. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Ch. 4.)

SOLUTION

Free-Body Diagram: Pulley ***A***



$$\xrightarrow{+} \Sigma F_x = 0: \quad -2P\left(\frac{5}{\sqrt{281}}\right) + P\cos\alpha = 0$$

$$\cos\alpha = 0.59655$$

$$\alpha = \pm 53.377^\circ$$

For $\alpha = +53.377^\circ$:

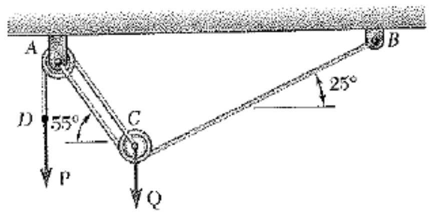
$$+\uparrow \Sigma F_y = 0: \quad 2P\left(\frac{16}{\sqrt{281}}\right) + P\sin 53.377^\circ - 1962\text{ N} = 0$$

$\mathbf{P} = 724\text{ N} \measuredangle 53.4^\circ$ ◀

For $\alpha = -53.377^\circ$:

$$+\uparrow \Sigma F_y = 0: \quad 2P\left(\frac{16}{\sqrt{281}}\right) + P\sin(-53.377^\circ) - 1962\text{ N} = 0$$

$\mathbf{P} = 1773 \; \nwarrow 53.4^\circ$ ◀

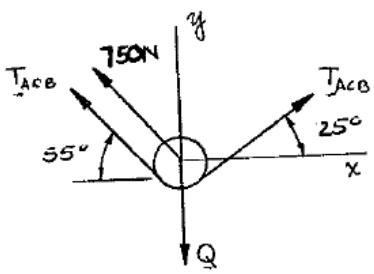


PROBLEM 2.69

A load **Q** is applied to the pulley C, which can roll on the cable ACB. The pulley is held in the position shown by a second cable CAD, which passes over the pulley A and supports a load **P**. Knowing that $P = 750$ N, determine (*a*) the tension in cable ACB, (*b*) the magnitude of load **Q**.

SOLUTION

Free-Body Diagram: Pulley ***C***



(*a*) $\xrightarrow{+} \Sigma F_x = 0{:}\quad T_{ACB}(\cos 25° - \cos 55°) - (750\text{ N})\cos 55° = 0$

Hence: $T_{ACB} = 1292.88$ N

$T_{ACB} = 1293$ N ◀

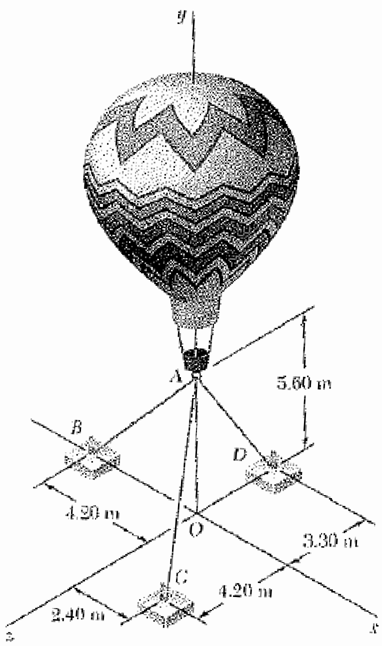
(*b*) $+\!\uparrow \Sigma F_y = 0{:}\quad T_{ACB}(\sin 25° + \sin 55°) + (750\text{ N})\sin 55° - Q = 0$

$(1292.88\text{ N})(\sin 25° + \sin 55°) + (750\text{ N})\sin 55° - Q = 0$

or $Q = 2219.8$ N $\qquad Q = 2220$ N ◀

PROBLEM 2.102

Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force at A, determine the tension in each cable.



SOLUTION

See Problem 2.101 for the figure and analysis leading to the linear algebraic Equations (1), (2), and (3).

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

From Eq. (1): $T_{AB} = 0.54053T_{AC}$

From Eq. (3): $T_{AD} = 1.11795T_{AC}$

Substituting for T_{AB} and T_{AD} in terms of T_{AC} into Eq. (2) gives

$$-0.8(0.54053T_{AC}) - 0.75676T_{AC} - 0.86154(1.11795T_{AC}) + P = 0$$

$$2.1523T_{AC} = P; \quad P = 800 \text{ N}$$

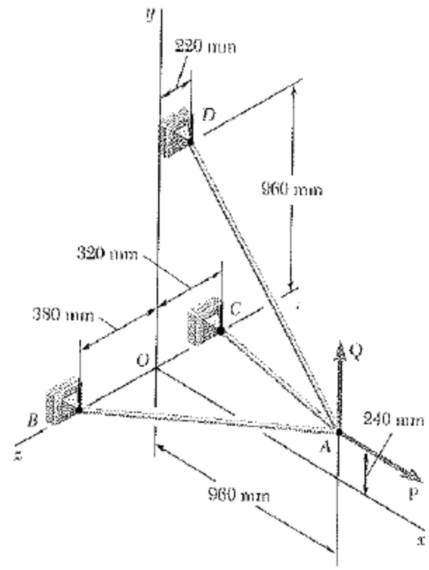
$$T_{AC} = \frac{800 \text{ N}}{2.1523} = 371.69 \text{ N}$$

Substituting into expressions for T_{AB} and T_{AD} gives

$$T_{AB} = 0.54053(371.69 \text{ N})$$

$$T_{AD} = 1.11795(371.69 \text{ N})$$

$T_{AB} = 201 \text{ N}, \quad T_{AC} = 372 \text{ N}, \quad T_{AD} = 416 \text{ N}$ ◀



PROBLEM 2.108

Three cables are connected at A, where the forces $\mathbf{P}$ and $\mathbf{Q}$ are applied as shown. Knowing that $P = 1200$ N, determine the values of Q for which cable AD is taut.

SOLUTION

We assume that $T_{AD} = 0$ and write $\quad \Sigma\mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + Q\mathbf{j} + (1200\text{ N})\mathbf{i} = 0$

$$\overrightarrow{AB} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} + (380\text{ mm})\mathbf{k} \quad AB = 1060\text{ mm}$$

$$\overrightarrow{AC} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \quad AC = 1040\text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)T_{AC}$$

Substituting into $\Sigma\mathbf{F}_A = 0$, factoring $\mathbf{i}, \mathbf{j}, \mathbf{k}$, and setting each coefficient equal to ϕ gives:

$$\mathbf{i}: \quad -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} + 1200\text{ N} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + Q = 0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 0 \qquad (3)$$

Solving the resulting system of linear equations using conventional algorithms gives:

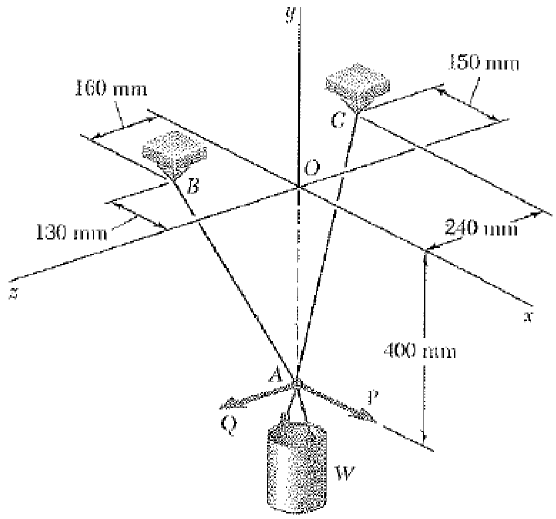
$$T_{AB} = 605.71\text{ N}$$

$$T_{AC} = 705.71\text{ N}$$

$$Q = 300.00\text{ N}$$

$0 \le Q < 300$ N ◀

Note: This solution assumes that Q is directed upward as shown $(Q \ge 0)$, if negative values of Q are considered, cable AD remains taut, but AC becomes slack for $Q = -460$ N.



PROBLEM 2.123

A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 376$ N, determine P and Q. (*Hint:* The tension is the same in both portions of cable BAC.)

SOLUTION

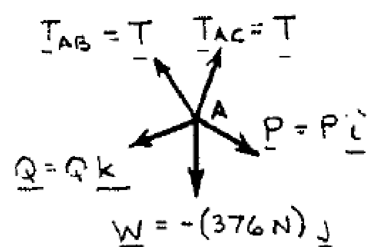
$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB}$$

$$= T\frac{\overrightarrow{AB}}{AB}$$

$$= T\frac{(-130 \text{ mm})\mathbf{i} + (400 \text{ mm})\mathbf{j} + (160 \text{ mm})\mathbf{k}}{450 \text{ mm}}$$

$$= T\left(-\frac{13}{45}\mathbf{i} + \frac{40}{45}\mathbf{j} + \frac{16}{45}\mathbf{k}\right)$$

Free-Body A:



$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC}$$

$$= T\frac{\overrightarrow{AC}}{AC}$$

$$= T\frac{(-150 \text{ mm})\mathbf{i} + (400 \text{ mm})\mathbf{j} + (-240 \text{ mm})\mathbf{k}}{490 \text{ mm}}$$

$$= T\left(-\frac{15}{49}\mathbf{i} + \frac{40}{49}\mathbf{j} - \frac{24}{49}\mathbf{k}\right)$$

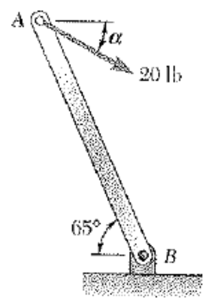
$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{Q} + \mathbf{P} + \mathbf{W} = 0$$

Setting coefficients of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \quad -\frac{13}{45}T - \frac{15}{49}T + P = 0 \qquad 0.59501T = P \qquad (1)$$

$$\mathbf{j}: \quad +\frac{40}{45}T + \frac{40}{49}T - W = 0 \qquad 1.70521T = W \qquad (2)$$

$$\mathbf{k}: \quad +\frac{16}{45}T - \frac{24}{49}T + Q = 0 \qquad 0.134240T = Q \qquad (3)$$

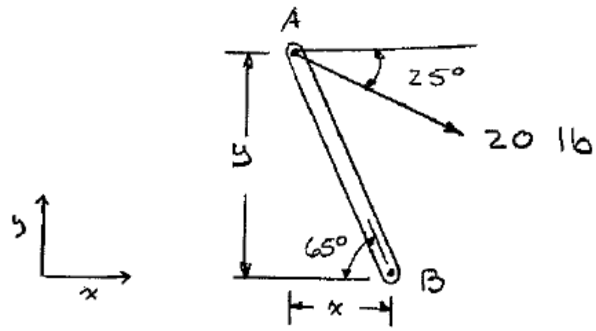


PROBLEM 3.1

A 20-lb force is applied to the control rod AB as shown. Knowing that the length of the rod is 9 in. and that $\alpha = 25°$, determine the moment of the force about Point B by resolving the force into horizontal and vertical components.

SOLUTION

Free-Body Diagram of Rod AB:



$$x = (9 \text{ in.})\cos 65° = 3.8036 \text{ in.}$$

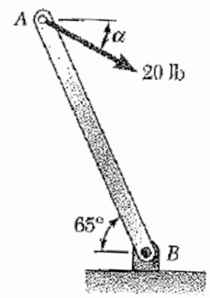
$$y = (9 \text{ in.})\sin 65° = 8.1568 \text{ in.}$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} = (20 \text{ lb} \cos 25°)\mathbf{i} + (-20 \text{ lb} \sin 25°)\mathbf{j} = (18.1262 \text{ lb})\mathbf{i} - (8.4524 \text{ lb})\mathbf{j}$$

$$\mathbf{r}_{A/B} = \overrightarrow{BA} = (-3.8036 \text{ in.})\mathbf{i} + (8.1568 \text{ in.})\mathbf{j}$$

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F} = (-3.8036\mathbf{i} + 8.1568\mathbf{j}) \times (18.1262\mathbf{i} - 8.4524\mathbf{j}) = 32.150\mathbf{k} - 147.852\mathbf{k} = -115.702 \text{ lb-in.}$$

$$\mathbf{M}_B = 115.7 \text{ lb-in.} \circlearrowright \blacktriangleleft$$

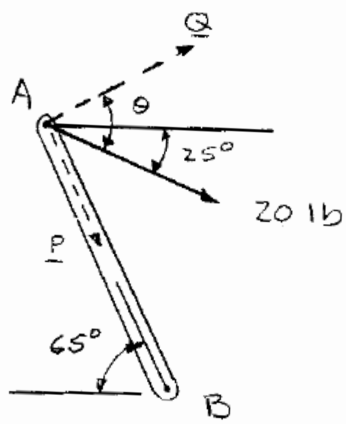


PROBLEM 3.2

A 20-lb force is applied to the control rod AB as shown. Knowing that the length of the rod is 9 in. and that $\alpha = 25°$, determine the moment of the force about Point B by resolving the force into components along AB and in a direction perpendicular to AB.

SOLUTION

Free-Body Diagram of Rod AB:

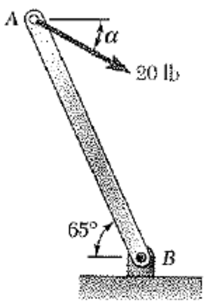


$$\theta = 90° - (65° - 25°)$$
$$= 50°$$

$$Q = (20 \text{ lb})\cos 50°$$
$$= 12.8558 \text{ lb}$$

$$M_B = Q(9 \text{ in.})$$
$$= (12.8558 \text{ lb})(9 \text{ in.})$$
$$= 115.702 \text{ lb-in.}$$

$\mathbf{M}_B = 115.7$ lb-in. ↻ ◀

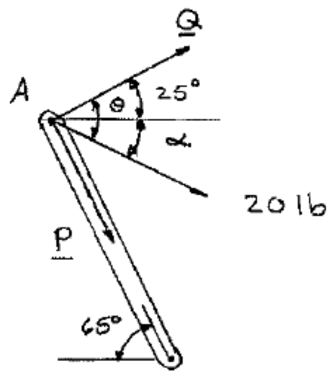


PROBLEM 3.3

A 20-lb force is applied to the control rod AB as shown. Knowing that the length of the rod is 9 in. and that the moment of the force about B is 120 lb · in. clockwise, determine the value of α.

SOLUTION

Free-Body Diagram of Rod AB:



$$\alpha = \theta - 25°$$

$$Q = (20\text{ lb})\cos\theta$$

and $$M_B = (Q)(9\text{ in.})$$

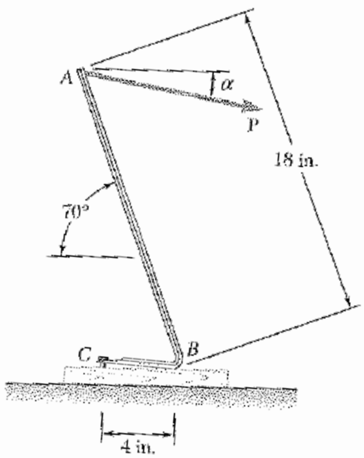
Therefore, $$120\text{ lb-in.} = (20\text{ lb})(\cos\theta)(9\text{ in.})$$

$$\cos\theta = \frac{120\text{ lb-in.}}{180\text{ lb-in.}}$$

or $$\theta = 48.190°$$

Therefore, $$\alpha = 48.190° - 25°$$

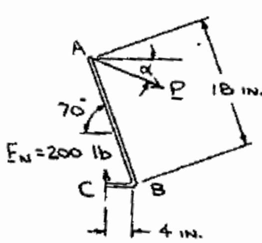
$\alpha = 23.2°$ ◀



PROBLEM 3.8

It is known that a vertical force of 200 lb is required to remove the nail at C from the board. As the nail first starts moving, determine (a) the moment about B of the force exerted on the nail, (b) the magnitude of the force **P** that creates the same moment about B if $\alpha = 10°$, (c) the smallest force **P** that creates the same moment about B.

SOLUTION

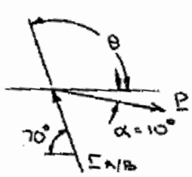


(a) We have $M_B = r_{C/B}F_N$

$= (4 \text{ in.})(200 \text{ lb})$

$= 800 \text{ lb}\cdot\text{in.}$

or $M_B = 800 \text{ lb}\cdot\text{in.}$ ↻ ◀



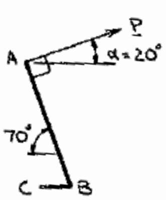
(b) By definition, $M_B = r_{A/B}P\sin\theta$

$\theta = 10° + (180° - 70°)$

$= 120°$

Then $800 \text{ lb}\cdot\text{in.} = (18 \text{ in.}) \times P\sin 120°$

or $P = 51.3 \text{ lb}$ ◀



(c) For **P** to be minimum, it must be perpendicular to the line joining Points A and B. Thus, **P** must be directed as shown.

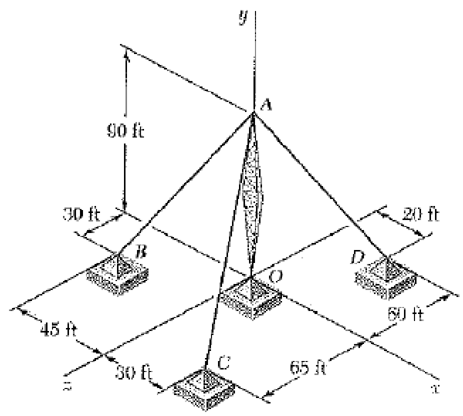
Thus $M_B = dP_{min}$

$d = r_{A/B}$

or $800 \text{ lb}\cdot\text{in.} = (18 \text{ in.})P_{min}$

or $P_{min} = 44.4 \text{ lb}$ $\mathbf{P}_{min} = 44.4 \text{ lb} \measuredangle 20°$ ◀

Bonus



PROBLEM 2.112

A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AC is 920 lb, determine the vertical force **P** exerted by the tower on the pin at A.

SOLUTION

See Problem 2.111 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{3}{7}T_{AB}+\frac{6}{23}T_{AC}+\frac{2}{11}T_{AD}=0 \quad (1)$$

$$-\frac{6}{7}T_{AB}-\frac{18}{23}T_{AC}-\frac{9}{11}T_{AD}+P=0 \quad (2)$$

$$\frac{2}{7}T_{AB}+\frac{13}{23}T_{AC}-\frac{6}{11}T_{AD}=0 \quad (3)$$

Substituting for $T_{AC}=920$ lb in Equations (1), (2), and (3) above and solving the resulting set of equations using conventional algorithms gives:

$$-\frac{3}{7}T_{AB}+240\text{ lb}+\frac{2}{11}T_{AD}=0 \quad (1')$$

$$-\frac{6}{7}T_{AB}-720\text{ lb}-\frac{9}{11}T_{AD}+P=0 \quad (2')$$

$$\frac{2}{7}T_{AB}+520\text{ lb}-\frac{6}{11}T_{AD}=0 \quad (3')$$

Solving,

$$T_{AB}=1240.00\text{ lb}$$
$$T_{AD}=1602.86\text{ lb}$$
$$P=3094.3\text{ lb}$$

$P=3090$ lb ◀