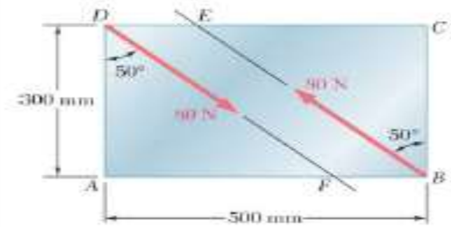
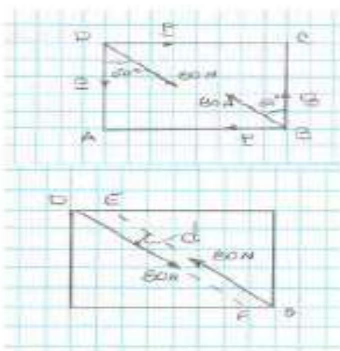
Homework #5 Solution



PROBLEM 3.70

Two 80-N forces are applied as shown to the corners B and D of a rectangular plate. (a) Determine the moment of the couple formed by the two forces by resolving each force into horizontal and vertical components and adding the moments of the two resulting couples. (b) Use the result obtained to determine the perpendicular distance between lines BE and DF.

SOLUTION



(a) Resolving forces into components:

$$P = (80 \text{ N})\sin 50^\circ = 61.284 \text{ N}$$

$$Q = (80 \text{ N})\cos 50^\circ = 51.423 \text{ N}$$

$$M = (51.423 \text{ N})(0.5 \text{ m}) - (61.284 \text{ N})(0.3 \text{ m})$$
$$= 7.3263 \text{ N}\cdot\text{m}$$

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$$\mathbf{M} = 7.33 \text{ N}\cdot\text{m} \circlearrowright$$ ◄

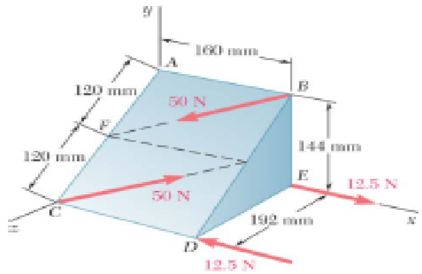
(b) Distance between lines BE and DF

$\mathbf{M} = Fd$

or $7.3263 \text{ N}\cdot\text{m} = (80 \text{ N})d$

$d = 0.091579 \text{ m}$

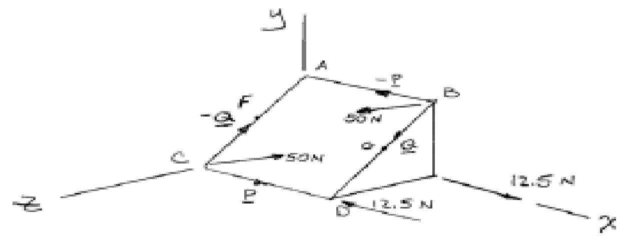
$d = 91.6 \text{ mm}$ ◄



PROBLEM 3.78

Replace the two couples shown with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Replace the couple in the *ABCD* plane with two couples *P* and *Q* shown:

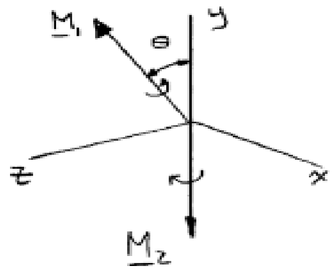
$$P = (50\text{ N})\frac{CD}{CG} = (50\text{ N})\left(\frac{160\text{ mm}}{200\text{ mm}}\right) = 40\text{ N}$$

$$Q = (50\text{ N})\frac{CF}{CG} = (50\text{ N})\left(\frac{120\text{ mm}}{200\text{ mm}}\right) = 30\text{ N}$$

Couple vector $\mathbf{M}_1$ perpendicular to plane *ABCD*:

$$+\circlearrowleft M_1 = (40\text{ N})(0.24\text{ m}) - (30\text{ N})(0.16\text{ m}) = 4.80\text{ N}\cdot\text{m}$$

Couple vector $\mathbf{M}_2$ in the *xy* plane:



$$+\circlearrowleft M_2 = -(12.5\text{ N})(0.192\text{ m}) = -2.40\text{ N}\cdot\text{m}$$

$$\tan\theta = \frac{144\text{ mm}}{192\text{ mm}} \qquad \theta = 36.870°$$

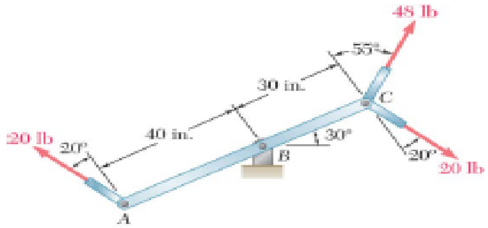
$$\mathbf{M}_1 = (4.80\cos 36.870°)\mathbf{j} + (4.80\sin 36.870°)\mathbf{k}$$
$$= 3.84\mathbf{j} + 2.88\mathbf{k}$$

$$\mathbf{M}_2 = -2.40\mathbf{j}$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 = 1.44\mathbf{j} + 2.88\mathbf{k}$$

$$\mathbf{M} = 3.22\text{ N}\cdot\text{m};\ \theta_x = 90.0°,\ \theta_y = 53.1°,\ \theta_z = 36.9° \blacktriangleleft$$

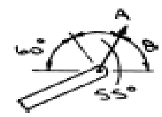
The angles in the solution are wrong (they don't take into account M_2. The correct answer is θ_x=90°, θ_y=63.4°, θ_z=26.6°



PROBLEM 3.89

Three control rods attached to a lever ABC exert on it the forces shown. (a) Replace the three forces with an equivalent force-couple system at B. (b) Determine the single force that is equivalent to the force-couple system obtained in Part a, and specify its point of application on the lever.

SOLUTION



(a) First note that the two 20-lb forces form A couple. Then

$$\mathbf{F} = 48\text{ lb} \measuredangle \theta$$

where $\theta = 180° - (60° + 55°) = 65°$

and

$$M = \Sigma M_B$$
$$= (30\text{ in.})(48\text{ lb})\cos 55° - (70\text{ in.})(20\text{ lb})\cos 20°$$
$$= -489.62\text{ lb}\cdot\text{in}$$

The equivalent force-couple system at B is

$$\mathbf{F} = 48.0\text{ lb} \measuredangle 65.0°; \qquad \mathbf{M} = 490\text{ lb}\cdot\text{in.} \curvearrowright$$ ◀

(b) The single equivalent force $\mathbf{F}'$ is equal to $\mathbf{F}$. Further, since the sense of $\mathbf{M}$ is clockwise, $\mathbf{F}'$ must be applied between A and B. For equivalence.

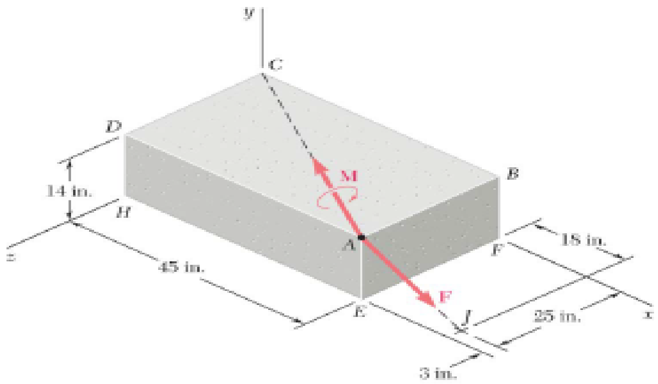
$$\Sigma M_B: \quad M = -aF'\cos 55°$$

where a is the distance from B to the point of application of $\mathbf{F}'$. Then

$$-489.62\text{ lb}\cdot\text{in.} = -a(48.0\text{ lb})\cos 55°$$

or $a = 17.78$ in. $\qquad \mathbf{F}' = 48.0\text{ lb} \measuredangle 65.0°$ ◀

and is applied to the lever 17.78 in. to the left of pin B ◀



PROBLEM 3.97

A 46-lb force **F** and a 2120-lb·in. couple **M** are applied to corner *A* of the block shown. Replace the given force-couple system with an equivalent force-couple system at corner *H*.

SOLUTION

We have $d_{AJ} = \sqrt{(18)^2 + (-14)^2 + (-3)^2} = 23 \text{ in.}$

Then $\mathbf{F} = \frac{46 \text{ lb}}{23}(18\mathbf{i} - 14\mathbf{j} - 3\mathbf{k})$

$= (36 \text{ lb})\mathbf{i} - (28 \text{ lb})\mathbf{j} - (6 \text{ lb})\mathbf{k}$

Also $d_{AC} = \sqrt{(-45)^2 + (0)^2 + (-28)^2} = 53 \text{ in.}$

Then $\mathbf{M} = \frac{2120 \text{ lb}\cdot\text{in.}}{53}(-45\mathbf{i} - 28\mathbf{k})$

$= -(1800 \text{ lb}\cdot\text{in.})\mathbf{i} - (1120 \text{ lb}\cdot\text{in.})\mathbf{k}$

Now $\mathbf{M}' = \mathbf{M} + \mathbf{r}_{A/H} \times \mathbf{F}$

where $\mathbf{r}_{A/H} = (45 \text{ in.})\mathbf{i} + (14 \text{ in.})\mathbf{j}$

Then $$\mathbf{M}' = (-1800\mathbf{i} - 1120\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 45 & 14 & 0 \\ 36 & -28 & -6 \end{vmatrix}$$

$= (-1800\mathbf{i} - 1120\mathbf{k}) + \{[(14)(-6)]\mathbf{i} + [-(45)(-6)]\mathbf{j} + [(45)(-28) - (14)(36)]\mathbf{k}\}$

$= (-1800 - 84)\mathbf{i} + (270)\mathbf{j} + (-1120 - 1764)\mathbf{k}$

$= -(1884 \text{ lb}\cdot\text{in.})\mathbf{i} + (270 \text{ lb}\cdot\text{in.})\mathbf{j} - (2884 \text{ lb}\cdot\text{in.})\mathbf{k}$

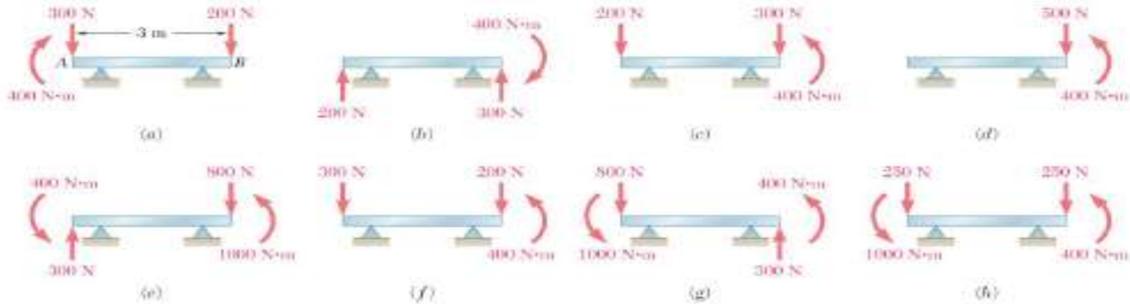
$= -(157 \text{ lb}\cdot\text{ft})\mathbf{i} + (22.5 \text{ lb}\cdot\text{ft})\mathbf{j} - (240 \text{ lb}\cdot\text{ft})\mathbf{k}$

The equivalent force-couple system at *H* is $\mathbf{F}' = (36.0 \text{ lb})\mathbf{i} - (28.0 \text{ lb})\mathbf{j} - (6.00 \text{ lb})\mathbf{k}$ ◀

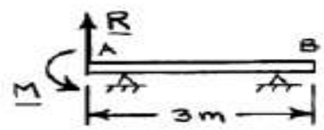
$\mathbf{M}' = -(157.0 \text{ lb}\cdot\text{ft})\mathbf{i} + (22.5 \text{ lb}\cdot\text{ft})\mathbf{j} - (240 \text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

PROBLEM 3.101

A 3-m-long beam is subjected to a variety of loadings. (*a*) Replace each loading with an equivalent force-couple system at end *A* of the beam. (*b*) Which of the loadings are equivalent?



SOLUTION



(*a*) (*a*) We have ΣF_y: $-300\text{ N} - 200\text{ N} = R_a$

or $\mathbf{R}_a = 500\text{ N}\downarrow$ ◀

and ΣM_A: $-400\text{ N}\cdot\text{m} - (200\text{ N})(3\text{ m}) = M_a$

or $\mathbf{M}_a = 1000\text{ N}\cdot\text{m}$ ↻ ◀

(*b*) We have ΣF_y: $200\text{ N} + 300\text{ N} = R_b$

or $\mathbf{R}_b = 500\text{ N}\uparrow$ ◀

and ΣM_A: $-400\text{ N}\cdot\text{m} + (300\text{ N})(3\text{ m}) = M_b$

or $\mathbf{M}_b = 500\text{ N}\cdot\text{m}$ ↺ ◀

(*c*) We have ΣF_y: $-200\text{ N} - 300\text{ N} = R_c$

or $\mathbf{R}_c = 500\text{ N}\downarrow$ ◀

and ΣM_A: $400\text{ N}\cdot\text{m} - (300\text{ N})(3\text{ m}) = M_c$

or $\mathbf{M}_c = 500\text{ N}\cdot\text{m}$ ↻ ◀

PROBLEM 3.101 (Continued)

(*d*) We have ΣF_y: $-500\text{ N} = R_d$

or $\mathbf{R}_d = 500\text{ N}\downarrow$ ◀

and ΣM_A: $400\text{ N}\cdot\text{m} - (500\text{ N})(3\text{ m}) = M_d$

or $\mathbf{M}_d = 1100\text{ N}\cdot\text{m}$ ↺ ◀

(*e*) We have ΣF_y: $300\text{ N} - 800\text{ N} = R_e$

or $\mathbf{R}_e = 500\text{ N}\downarrow$ ◀

and ΣM_A: $400\text{ N}\cdot\text{m} + 1000\text{ N}\cdot\text{m} - (800\text{ N})(3\text{ m}) = M_e$

or $\mathbf{M}_e = 1000\text{ N}\cdot\text{m}$ ↺ ◀

(*f*) We have ΣF_y: $-300\text{ N} - 200\text{ N} = R_f$

or $\mathbf{R}_f = 500\text{ N}\downarrow$ ◀

and ΣM_A: $400\text{ N}\cdot\text{m} - (200\text{ N})(3\text{ m}) = M_f$

or $\mathbf{M}_f = 200\text{ N}\cdot\text{m}$ ↺ ◀

(*g*) We have ΣF_y: $-800\text{ N} + 300\text{ N} = R_g$

or $\mathbf{R}_g = 500\text{ N}\downarrow$ ◀

and ΣM_A: $1000\text{ N}\cdot\text{m} + 400\text{ N}\cdot\text{m} + (300\text{ N})(3\text{ m}) = M_g$

or $\mathbf{M}_g = 2300\text{ N}\cdot\text{m}$ ↻ ◀

(*h*) We have ΣF_y: $-250\text{ N} - 250\text{ N} = R_h$

or $\mathbf{R}_h = 500\text{ N}\downarrow$ ◀

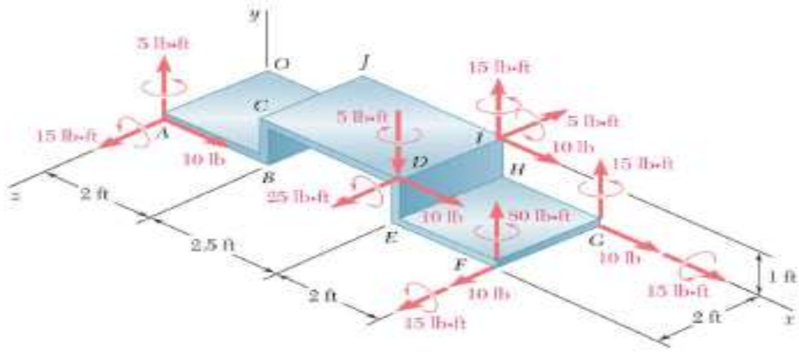
and ΣM_A: $1000\text{ N}\cdot\text{m} + 400\text{ N}\cdot\text{m} - (250\text{ N})(3\text{ m}) = M_h$

or $\mathbf{M}_h = 650\text{ N}\cdot\text{m}$ ↻ ◀

(*b*) Therefore, loadings (*a*) and (*e*) are equivalent.

PROBLEM 3.104

Five separate force-couple systems act at the corners of a piece of sheet metal, which has been bent into the shape shown. Determine which of these systems is equivalent to a force $\mathbf{F} = (10 \text{ lb})\mathbf{i}$ and a couple of moment $\mathbf{M} = (15 \text{ lb}\cdot\text{ft})\mathbf{j} + (15 \text{ lb}\cdot\text{ft})\mathbf{k}$ located at the origin.



SOLUTION

First note that the force-couple system at F cannot be equivalent because of the direction of the force [The force of the other four systems is (10 lb)**i**]. Next, move each of the systems to the origin O; the forces remain unchanged.

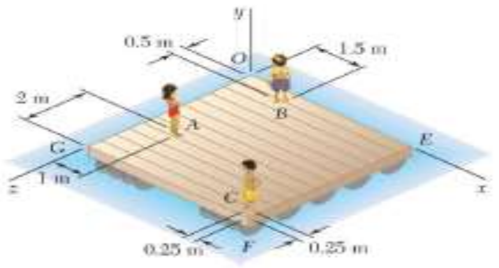
$$A: \quad \mathbf{M}_A = \Sigma\mathbf{M}_O = (5 \text{ lb}\cdot\text{ft})\mathbf{j} + (15 \text{ lb}\cdot\text{ft})\mathbf{k} + (2 \text{ ft})\mathbf{k} \times (10 \text{ lb})\mathbf{i}$$
$$= (25 \text{ lb}\cdot\text{ft})\mathbf{j} + (15 \text{ lb}\cdot\text{ft})\mathbf{k}$$

$$D: \quad \mathbf{M}_D = \Sigma\mathbf{M}_O = -(5 \text{ lb}\cdot\text{ft})\mathbf{j} + (25 \text{ lb}\cdot\text{ft})\mathbf{k}$$
$$+ [(4.5 \text{ ft})\mathbf{i} + (1 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}] \times 10 \text{ lb})\mathbf{i}$$
$$= (15 \text{ lb}\cdot\text{ft})\mathbf{i} + (15 \text{ lb}\cdot\text{ft})\mathbf{k}$$

$$G: \quad \mathbf{M}_G = \Sigma\mathbf{M}_O = (15 \text{ lb}\cdot\text{ft})\mathbf{i} + (15 \text{ lb}\cdot\text{ft})\mathbf{j}$$

$$I: \quad \mathbf{M}_I = \Sigma\mathbf{M}_I = (15 \text{ lb}\cdot\text{ft})\mathbf{j} - (5 \text{ lb}\cdot\text{ft})\mathbf{k}$$
$$+ [(4.5 \text{ ft})\mathbf{i} + (1 \text{ ft})\mathbf{j}] \times (10 \text{ lb})\mathbf{j}$$
$$= (15 \text{ lb}\cdot\text{ft})\mathbf{j} - (15 \text{ lb}\cdot\text{ft})\mathbf{k}$$

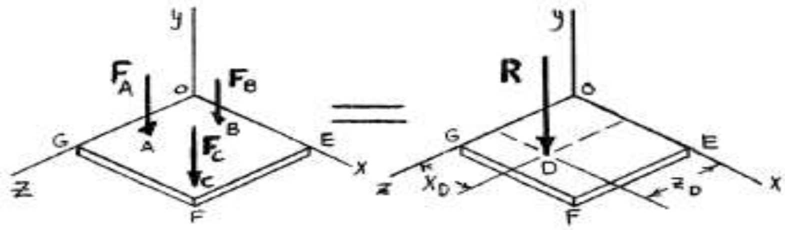
The equivalent force-couple system is the system at corner D. ◀



PROBLEM 3.127

Three children are standing on a 5×5-m raft. If the weights of the children at Points A, B, and C are 375 N, 260 N, and 400 N, respectively, determine the magnitude and the point of application of the resultant of the three weights.

SOLUTION



We have $\quad \Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{R}$

$$-(375\text{ N})\mathbf{j} - (260\text{ N})\mathbf{j} - (400\text{ N})\mathbf{j} = \mathbf{R}$$

$$-(1035\text{ N})\mathbf{j} = \mathbf{R}$$

or $\quad R = 1035\text{ N}$ ◀

We have $\quad \Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) = R(z_D)$

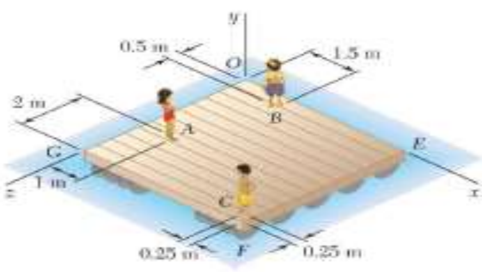
$$(375\text{ N})(3\text{ m}) + (260\text{ N})(0.5\text{ m}) + (400\text{ N})(4.75\text{ m}) = (1035\text{ N})(z_D)$$

$z_D = 3.0483\text{ m}$ $\quad$ or $\quad z_D = 3.05\text{ m}$ ◀

We have $\quad \Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) = R(x_D)$

$$375\text{ N}(1\text{ m}) + (260\text{ N})(1.5\text{ m}) + (400\text{ N})(4.75\text{ m}) = (1035\text{ N})(x_D)$$

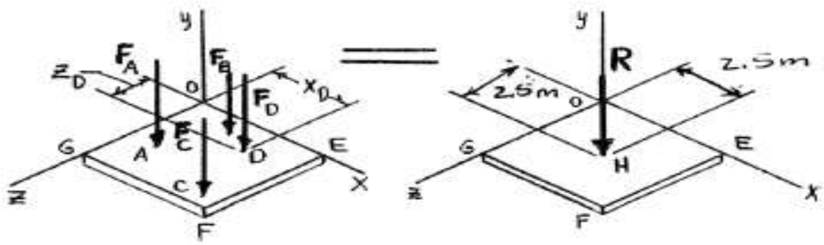
$x_D = 2.5749\text{ m}$ $\quad$ or $\quad x_D = 2.57\text{ m}$ ◀



PROBLEM 3.128

Three children are standing on a 5×5-m raft. The weights of the children at Points *A*, *B*, and *C* are 375 N, 260 N, and 400 N, respectively. If a fourth child of weight 425 N climbs onto the raft, determine where she should stand if the other children remain in the positions shown and the line of action of the resultant of the four weights is to pass through the center of the raft.

SOLUTION



We have $\quad \Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{R}$

$$-(375\text{ N})\mathbf{j} - (260\text{ N})\mathbf{j} - (400\text{ N})\mathbf{j} - (425\text{ N})\mathbf{j} = \mathbf{R}$$

$$\mathbf{R} = -(1460\text{ N})\mathbf{j}$$

We have $\quad \Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) = R(z_H)$

$$(375\text{ N})(3\text{ m}) + (260\text{ N})(0.5\text{ m}) + (400\text{ N})(4.75\text{ m}) + (425\text{ N})(z_D) = (1460\text{ N})(2.5\text{ m})$$

$z_D = 1.16471\text{ m}$ $\qquad$ or $\quad z_D = 1.165\text{ m}$ ◀

We have $\quad \Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) = R(x_H)$

$$(375\text{ N})(1\text{ m}) + (260\text{ N})(1.5\text{ m}) + (400\text{ N})(4.75\text{ m}) + (425\text{ N})(x_D) = (1460\text{ N})(2.5\text{ m})$$

$x_D = 2.3235\text{ m}$ $\qquad$ or $\quad x_D = 2.32\text{ m}$ ◀