Homework #5 Solution

PROBLEM 3.70

A plate in the shape of a parallelogram is acted upon by two couples. Determine (a) the moment of the couple formed by the two 21-lb forces, (b) the perpendicular distance between the 12-lb forces if the resultant of the two couples is zero, (c) the value of $\alpha$ if the resultant couple is 72 lb·in. clockwise and $d$ is 42 in.

SOLUTION

(a) We have $M_1 = d_1 F_1$

where $d_1 = 16$ in.
$F_1 = 21$ lb

$M_1 = (16 \text{ in.})(21 \text{ lb}) = 336 \text{ lb·in.}$

or $M_1 = 336 \text{ lb·in.}$

(b) We have $M_1 + M_2 = 0$

or $336 \text{ lb·in.} - d_2(12 \text{ lb}) = 0$

$d_2 = 28.0$ in.

(c) We have $M_{\text{total}} = M_1 + M_2$

or $-72 \text{ lb·in.} = 336 \text{ lb·in.} - (42 \text{ in.})(\sin \alpha)(12 \text{ lb})$

$\sin \alpha = 0.80952$

and $\alpha = 54.049^\circ$ or $\alpha = 54.0^\circ$
PROBLEM 3.76

Replace the two couples shown with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

Replace the couple in the $ABCD$ plane with two couples $P$ and $Q$ shown:

\[ P = (50 \text{ N}) \left( \frac{CD}{CG} \right) = (50 \text{ N}) \left( \frac{160 \text{ mm}}{200 \text{ mm}} \right) = 40 \text{ N} \]

\[ Q = (50 \text{ N}) \left( \frac{CF}{CG} \right) = (50 \text{ N}) \left( \frac{120 \text{ mm}}{200 \text{ mm}} \right) = 30 \text{ N} \]

Couple vector $M_1$ perpendicular to plane $ABCD$:

\[ + M_1 = (40 \text{ N})(0.24 \text{ m}) - (30 \text{ N})(0.16 \text{ m}) = 4.80 \text{ N} \cdot \text{m} \]

Couple vector $M_2$ in the $xy$ plane:

\[ + M_2 = -(12.5 \text{ N})(0.192 \text{ m}) = -2.40 \text{ N} \cdot \text{m} \]

\[ \tan \theta = \frac{144 \text{ mm}}{192 \text{ mm}} \Rightarrow \theta = 36.870^\circ \]

\[ M_1 = (4.80 \cos 36.870^\circ) \hat{j} + (4.80 \sin 36.870^\circ) \hat{k} = 3.84 \hat{j} + 2.88 \hat{k} \]

\[ M_2 = -2.40 \hat{j} \]

\[ M = M_1 + M_2 = 1.44 \hat{j} + 2.88 \hat{k} \]

\[ M = 3.22 \text{ N} \cdot \text{m}; \theta_x = 90.0^\circ, \theta_y = 53.1^\circ, \theta_z = 36.9^\circ \]
**PROBLEM 3.87**

Three control rods attached to a lever ABC exert on it the forces shown. (a) Replace the three forces with an equivalent force-couple system at B. (b) Determine the single force that is equivalent to the force-couple system obtained in part a, and specify its point of application on the lever.

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**SOLUTION**

(a) First note that the two 90-N forces form a couple. Then

\[ \mathbf{F} = 216 \text{ N} \ \theta \]

where

\[ \theta = 180^\circ - (60^\circ + 55^\circ) = 65^\circ \]

and

\[ M = \sum M_y = (0.450 \text{ m})(216 \text{ N}) \cos 55^\circ - (1.050 \text{ m})(90 \text{ N}) \cos 20^\circ \]

\[ = -33.049 \text{ N} \cdot \text{m} \]

The equivalent force-couple system at B is:

\[ \mathbf{F} = 216 \text{ N} \ \theta = 65.0^\circ; \ M = 33.0 \text{ N} \cdot \text{m} \]

(b) The single equivalent force \( \mathbf{F}' \) is equal to \( \mathbf{F} \). Further, since the sense of \( M \) is clockwise, \( \mathbf{F}' \) must be applied between A and B. For equivalence,

\[ \Sigma M_B: \ M = \alpha \mathbf{F}' \cos 55^\circ \]

where \( \alpha \) is the distance from B to the point of application of \( \mathbf{F}' \). Then

\[ -33.049 \text{ N} \cdot \text{m} = -\alpha (216 \text{ N}) \cos 55^\circ \]

\[ \alpha = 0.26676 \text{ m} \]

or

\[ \mathbf{F}' = 216 \text{ N} \ \theta = 65.0^\circ \text{ applied to the lever 267 mm to the left of } B \]

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**PROBLEM 3.98**

A 46-lb force \( F \) and a 2120-lb-in. couple \( M \) are applied to corner \( A \) of the block shown. Replace the given force-couple system with an equivalent force-couple system at corner \( H \).

**SOLUTION**

We have

\[
d_{AJ} = \sqrt{(18)^2 + (-14)^2 + (-3)^2} = 23 \text{ in.}
\]

Then

\[
F = \frac{46 \text{ lb}}{23} (18\hat{i} - 14\hat{j} - 3\hat{k}) \\
= (36 \text{ lb})\hat{i} - (28 \text{ lb})\hat{j} - (6 \text{ lb})\hat{k}
\]

Also

\[
d_{AC} = \sqrt{(-45)^2 + (0)^2 + (-28)^2} = 53 \text{ in.}
\]

Then

\[
M = \frac{2120 \text{ lb-in.}}{53} (-45\hat{i} - 28\hat{k}) \\
= -(1800 \text{ lb-in.})\hat{i} - (1120 \text{ lb-in.})\hat{k}
\]

Now

\[
M' = M + r_{AJ} \times F
\]

where

\[
r_{AJ} = (45 \text{ in.})\hat{i} + (14 \text{ in.})\hat{j}
\]

Then

\[
M' = (-1800\hat{i} - 1120\hat{k}) + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 45 & 14 & 0 \\ 36 & -28 & -6 \end{vmatrix}
\]

\[
= (-1800\hat{i} - 1120\hat{k}) + [(45)(14)(0) + (45)(-60) + (1120)(-28) - (1800)(36)]\hat{k}
\]

\[
= (-1800\hat{i} - 1120\hat{k}) + [0 - (45)(-60) + (-1120)(28) - (1800)(36)]\hat{k}
\]

\[
= (-1800\hat{i} - 1120\hat{k}) + [(-1800)(-84) + (270)(-1764)]\hat{k}
\]

\[
= (-1884 \text{ lb-in.})\hat{i} + (270 \text{ lb-in.})\hat{j} - (2884 \text{ lb-in.})\hat{k}
\]

\[
= (-157 \text{ lb-ft})\hat{i} + (22.5 \text{ lb-ft})\hat{j} - (240 \text{ lb-ft})\hat{k}
\]

The equivalent force-couple system at \( H \) is

\[
F' = (36.0 \text{ lb})\hat{i} - (28.0 \text{ lb})\hat{j} - (6.00 \text{ lb})\hat{k}
\]

\[
M' = (157.0 \text{ lb-ft})\hat{i} + (22.5 \text{ lb-ft})\hat{j} - (240 \text{ lb-ft})\hat{k}
\]
PROBLEM 3.101

A 3-m-long beam is subjected to a variety of loadings. (a) Replace each loading with an equivalent force-couple system at end A of the beam. (b) Which of the loadings are equivalent?

(a) We have \[ \Sigma F_y: \quad -300 \, N - 200 \, N = R_a \]

and \[ \Sigma M_A: \quad -400 \, N \cdot m - (200 \, N)(3 \, m) = M_a \]

or \[ R_a = 500 \, N \uparrow \]

or \[ M_a = 1000 \, N \cdot m \uparrow \]

(b) We have \[ \Sigma F_y: \quad 200 \, N + 300 \, N = R_b \]

and \[ \Sigma M_A: \quad -400 \, N \cdot m + (300 \, N)(3 \, m) = M_b \]

or \[ R_b = 500 \, N \uparrow \]

or \[ M_b = 500 \, N \cdot m \uparrow \]

(c) We have \[ \Sigma F_y: \quad -200 \, N - 300 \, N = R_c \]

and \[ \Sigma M_A: \quad 400 \, N \cdot m - (300 \, N)(3 \, m) = M_c \]

or \[ M_c = 500 \, N \cdot m \uparrow \]

or \[ R_c = 500 \, N \uparrow \]
PROBLEM 3.104

Five separate force-couple systems act at the corners of a piece of sheet metal, which has been bent into the shape shown. Determine which of these systems is equivalent to a force \( \mathbf{F} = (10 \text{ lb})\mathbf{i} \) and a couple of moment \( \mathbf{M} = (15 \text{ lb} \cdot \text{ft})\mathbf{j} + (15 \text{ lb} \cdot \text{ft})\mathbf{k} \) located at the origin.

**SOLUTION**

First note that the force-couple system at \( F \) cannot be equivalent because of the direction of the force [The force of the other four systems is \( (10 \text{ lb})\mathbf{i} \)]. Next, move each of the systems to the origin \( O \); the forces remain unchanged.

\[
A: \quad \mathbf{M}_A = \sum \mathbf{M}_D = (5 \text{ lb} \cdot \text{ft})\mathbf{j} + (15 \text{ lb} \cdot \text{ft})\mathbf{k} + (2 \text{ ft})\mathbf{k} \times (10 \text{ lb})\mathbf{i} \\
\quad \quad \quad \quad \quad = (25 \text{ lb} \cdot \text{ft})\mathbf{j} + (15 \text{ lb} \cdot \text{ft})\mathbf{k}
\]

\[
D: \quad \mathbf{M}_D = \sum \mathbf{M}_O = -(5 \text{ lb} \cdot \text{ft})\mathbf{j} + (25 \text{ lb} \cdot \text{ft})\mathbf{k} \\
\quad \quad \quad \quad \quad + [(4.5 \text{ ft})\mathbf{i} + (1 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}] \times (10 \text{ lb})\mathbf{i} \\
\quad \quad \quad \quad \quad = (15 \text{ lb} \cdot \text{ft})\mathbf{i} + (15 \text{ lb} \cdot \text{ft})\mathbf{k}
\]

\[
G: \quad \mathbf{M}_G = \sum \mathbf{M}_O = (15 \text{ lb} \cdot \text{ft})\mathbf{i} + (15 \text{ lb} \cdot \text{ft})\mathbf{j}
\]

\[
I: \quad \mathbf{M}_I = \sum \mathbf{M}_I = (15 \text{ lb} \cdot \text{ft})\mathbf{j} - (5 \text{ lb} \cdot \text{ft})\mathbf{k} \\
\quad \quad \quad \quad \quad + [(4.5 \text{ ft})\mathbf{i} + (1 \text{ ft})\mathbf{j}] \times (10 \text{ lb})\mathbf{j} \\
\quad \quad \quad \quad \quad = (15 \text{ lb} \cdot \text{ft})\mathbf{j} - (15 \text{ lb} \cdot \text{ft})\mathbf{k}
\]

The equivalent force-couple system is the system at corner \( D \).
PROBLEM 3.127

Three children are standing on a 5×5-m raft. If the weights of the children at Points A, B, and C are 375 N, 260 N, and 400 N, respectively, determine the magnitude and the point of application of the resultant of the three weights.

SOLUTION

We have

\[ \sum F: \quad F_A + F_B + F_C = R \]

\[ -(375 \text{ N})j - (260 \text{ N})j - (400 \text{ N})j = R \]

\[ -(1035 \text{ N})j = R \]

or \( R = 1035 \text{ N} \) ▲

We have

\[ \sum M_z: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) = R(z_D) \]

\[ (375 \text{ N})(3 \text{ m}) + (260 \text{ N})(0.5 \text{ m}) + (400 \text{ N})(4.75 \text{ m}) = (1035 \text{ N})(z_D) \]

\[ z_D = 3.0483 \text{ m} \]

or \( z_D = 3.05 \text{ m} \) ▲

We have

\[ \sum M_y: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) = R(x_D) \]

\[ 375 \text{ N}(1 \text{ m}) + (260 \text{ N})(1.5 \text{ m}) + (400 \text{ N})(4.75 \text{ m}) = (1035 \text{ N})(x_D) \]

\[ x_D = 2.5749 \text{ m} \]

or \( x_D = 2.57 \text{ m} \) ▲
PROBLEM 3.128

Three children are standing on a 5×5-m raft. The weights of the children at Points A, B, and C are 375 N, 260 N, and 400 N, respectively. If a fourth child of weight 425 N climbs onto the raft, determine where she should stand if the other children remain in the positions shown and the line of action of the resultant of the four weights is to pass through the center of the raft.

SOLUTION

We have

\[ \Sigma F: \quad F_A + F_B + F_C + F_D = R \]

\[ -(375 \text{ N})j + (260 \text{ N})j + (400 \text{ N})j + (425 \text{ N})j = R \]

\[ R = -(1460 \text{ N})j \]

We have

\[ \Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) = R(z_H) \]

\[ 375 \text{ N}(3 \text{ m}) + 260 \text{ N}(0.5 \text{ m}) + 400 \text{ N}(4.75 \text{ m}) \]

\[ + 425 \text{ N}(z_D) = 1460 \text{ N}(2.5 \text{ m}) \]

\[ z_D = 1.16471 \text{ m} \]

or \[ z_D = 1.165 \text{ m} \]

We have

\[ \Sigma M_y: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) = R(x_H) \]

\[ 375 \text{ N}(1 \text{ m}) + 260 \text{ N}(1.5 \text{ m}) + 400 \text{ N}(4.75 \text{ m}) \]

\[ + 425 \text{ N}(x_D) = 1460 \text{ N}(2.5 \text{ m}) \]

\[ x_D = 2.3235 \text{ m} \]

or \[ x_D = 2.32 \text{ m} \]