**MECH 223 – Engineering Statics**

**Midterm 2, April 14th 2015**

**Question 1 (20 + 5 points)**

(a) (10 points) In each of the two following examples, in the column to the right, draw a free body diagram (FBD) of the body to be isolated, shown in the middle column. Dimensions and numerical values are omitted for simplicity.

<table>
<thead>
<tr>
<th>Description</th>
<th>Body to be Isolated</th>
<th>FBD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform pole of mass $m$ hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of the pole.</td>
<td><img src="image1" alt="Notch" /></td>
<td></td>
</tr>
<tr>
<td>Uniform horizontal bar of mass $m$ suspended by vertical cable at $A$ and supported by a smooth surface at $B$</td>
<td><img src="image2" alt="Cable" /></td>
<td></td>
</tr>
<tr>
<td>Uniform grooved wheel of mass $m$ supported by a rough surface and by action of a horizontal cable</td>
<td><img src="image3" alt="Wheel" /></td>
<td></td>
</tr>
<tr>
<td>Uniform heavy plate of mass $m$ supported in vertical plane by cable $C$ and hinge $A$</td>
<td><img src="image4" alt="Plate" /></td>
<td></td>
</tr>
</tbody>
</table>
(b) (15 points) In the following 5 sketches determine whether (a) the body is completely, partially or improperly constrained, (b) the reactions are statically determinate or indeterminate, (c) the equilibrium of the body is maintained in the position shown. All connections in cases 1-4 consist of frictionless pins, rollers or short links. In case 5 the surfaces are smooth. Explain your answers. Draw any sketches/FBDs needed.
**Question 2 (20 points)** A Pratt roof truss is loaded as shown in the figure to the right.

(a) (10 points) Neglect any horizontal reactions at the supports and solve for the forces in all the members if $P = Q = 1 \text{ kN}$. 
(b) (5 points) If $P = 0 \text{kN}$, and $Q = 4 \text{kN}$, what are the forces in members $AB$ and $AH$? Explain.

(e) (5 points) Under the loading condition in part (b) identify by inspection those members in which the forces are zero. Explain.
Question 3 (30 points)

(a) (15 points) Determine the coordinates of the centroid of the bracket in the figure to the right (use the attached tables of centroids of common bodies).
(b) (10 points) Determine by integration the y coordinate of the centroid of the shaded area in the figure to the right. **Solutions by other methods will not carry any credit!**

(c) (5 points) By treating the shaded area in part (b) as a **composite body**, find the x coordinate of the centroid. Use the attached tables of centroids of common bodies.
**Question 3 (30 + 10 points)** The 20 X 20-in square plate weighs 56 lb and is supported by three vertical wires as shown in the figure to the right.

(a) (13 points) Determine the tension in each wire.

(b) (12 points) If a vertical force of 10 lb is applied down on the plate, what is the point of application if it results in all tensions being equal (8 points)? What is the corresponding tension (4)?
(c) (5 points) What is the maximum distance \( d \) from the \( z \) axis for a 150 lb force down to be applied on the plate without the plate tipping?

(d) **Bonus** (10 points) Show on a sketch the area of the plate over which the force in part (c) can act without the plate tipping. Mark relevant distances.

Show your work!

Good Luck!
Centroids of Common 1D Bodies

<table>
<thead>
<tr>
<th>Shape</th>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter-circular arc</td>
<td>$\frac{2r}{\pi}$</td>
<td>$\frac{2r}{\pi}$</td>
<td>$\frac{\pi r}{2}$</td>
</tr>
<tr>
<td>Semicircular arc</td>
<td>0</td>
<td>$\frac{2r}{\pi}$</td>
<td>$\pi r$</td>
</tr>
<tr>
<td>Arc of circle</td>
<td>$r \sin \alpha / \alpha$</td>
<td>0</td>
<td>$2\alpha r$</td>
</tr>
</tbody>
</table>

Fig. 5.8B  Centroids of common shapes of lines.

Centroids of Common 2D Bodies

<table>
<thead>
<tr>
<th>Shape</th>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular area</td>
<td></td>
<td>$\frac{h}{3}$</td>
<td>$\frac{bh}{2}$</td>
</tr>
<tr>
<td>Quarter-circular area</td>
<td>$\frac{4r}{3\pi}$</td>
<td>$\frac{4r}{3\pi}$</td>
<td>$\frac{\pi r^2}{4}$</td>
</tr>
<tr>
<td>Semicircular area</td>
<td>0</td>
<td>$\frac{4r}{3\pi}$</td>
<td>$\frac{\pi r^2}{2}$</td>
</tr>
<tr>
<td>Quarter-elliptical area</td>
<td>$\frac{4a}{3\pi}$</td>
<td>$\frac{4b}{3\pi}$</td>
<td>$\frac{\pi ab}{4}$</td>
</tr>
<tr>
<td>Semielliptical area</td>
<td>0</td>
<td>$\frac{4b}{3\pi}$</td>
<td>$\frac{\pi ab}{2}$</td>
</tr>
<tr>
<td>Semiparabolic area</td>
<td>$\frac{3a}{8}$</td>
<td>$\frac{3h}{5}$</td>
<td>$\frac{2ah}{3}$</td>
</tr>
<tr>
<td>Parabolic area</td>
<td>0</td>
<td>$\frac{3h}{5}$</td>
<td>$\frac{4ah}{3}$</td>
</tr>
</tbody>
</table>
### Centroids of Common 3D Bodies

<table>
<thead>
<tr>
<th>Shape</th>
<th>( \bar{x} )</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hemisphere</td>
<td>( \frac{3a}{8} )</td>
<td>( \frac{2}{3} \pi a^3 )</td>
</tr>
<tr>
<td>Semiellipsoid of revolution</td>
<td>( \frac{3h}{8} )</td>
<td>( \frac{2}{3} \pi a^2 h )</td>
</tr>
<tr>
<td>Shape</td>
<td>Formula</td>
<td>$h/3$</td>
</tr>
<tr>
<td>------------------------</td>
<td>--------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>Paraboloid of revolution</td>
<td></td>
<td>$h/4$</td>
</tr>
<tr>
<td>Cone</td>
<td></td>
<td>$h/4$</td>
</tr>
<tr>
<td>Pyramid</td>
<td></td>
<td>$h/4$</td>
</tr>
</tbody>
</table>

Fig. 5.21 Centroids of common shapes and volumes.