1. Locate the center of mass of the bracket-and-shaft combination. The vertical face is made from sheet metal which has a mass of 25 kg/m². The material of the horizontal base has a mass of 40 kg/m², and the steel shaft has a density of 7.83 Mg/m³.

Sample Problem 5/8

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Solution. The composite body may be considered to be composed of the five elements shown in the lower portion of the illustration. The triangular part will be taken as a negative mass. For the reference axes indicated it is clear by symmetry that the x-coordinate of the center of mass is zero.

The mass \( m \) of each part is easily calculated and should need no further explanation. For Part 1 we have from Sample Problem 5/3
\[
\bar{z} = \frac{4r}{3\pi} = \frac{4(50)}{3\pi} = 21.2 \text{ mm}
\]

For Part 3 from Sample Problem 5/2 we see that the centroid of the triangular mass is one-third of its altitude above its base. Measurement from the coordinate axes becomes
\[
\bar{z} = -[150 - 25 - \frac{1}{4}(75)] = -100 \text{ mm}
\]

The \( y \) - and \( z \)-coordinates to the mass centers of the remaining parts should be evident by inspection. The terms involved in applying Eqs. 6/7 are best handled in the form of a table as follows:

<table>
<thead>
<tr>
<th>PART</th>
<th>( m ) kg</th>
<th>( \bar{y} ) mm</th>
<th>( \bar{z} ) mm</th>
<th>( m\bar{y} ) kg·mm</th>
<th>( m\bar{z} ) kg·mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.098</td>
<td>0</td>
<td>21.2</td>
<td>0</td>
<td>2.08</td>
</tr>
<tr>
<td>2</td>
<td>0.562</td>
<td>0</td>
<td>-75.0</td>
<td>0</td>
<td>-42.19</td>
</tr>
<tr>
<td>3</td>
<td>-0.084</td>
<td>0</td>
<td>-100.0</td>
<td>0</td>
<td>9.38</td>
</tr>
<tr>
<td>4</td>
<td>0.600</td>
<td>50.0</td>
<td>-150.0</td>
<td>30.0</td>
<td>-90.00</td>
</tr>
<tr>
<td>5</td>
<td>1.476</td>
<td>75.0</td>
<td>0</td>
<td>110.7</td>
<td>0</td>
</tr>
<tr>
<td>TOTALS</td>
<td>2.642</td>
<td>140.7</td>
<td>-120.73</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Equations 5/7 are now applied and the results are
\[
\bar{y} = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{140.7}{2.642} = 53.3 \text{ mm} \quad \text{Ans.}
\]
\[
\bar{z} = \frac{\Sigma m\bar{z}}{\Sigma m} = \frac{-120.73}{2.642} = -45.7 \text{ mm} \quad \text{Ans.}
\]
2. The welded assembly is made of a uniform rod weighing 0.370 lb per foot of length and the semi-circular plate weighing 8 lb per square foot. Calculate the coordinates of the center of gravity of the assembly.

Solution:

<table>
<thead>
<tr>
<th>Port</th>
<th>L, ft</th>
<th>A, ft²</th>
<th>W, lb</th>
<th>Z, in.</th>
<th>WR</th>
<th>W²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.047</td>
<td></td>
<td>0.387</td>
<td>6</td>
<td>2.325</td>
<td>0.987</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-</td>
<td>0.370</td>
<td>3</td>
<td>1.110</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>0.1745</td>
<td>1.396</td>
<td>1.698</td>
<td>2.370</td>
<td></td>
</tr>
<tr>
<td>Σ</td>
<td>5.63</td>
<td>2.153</td>
<td></td>
<td>3.435</td>
<td>1.595 in</td>
<td>3.357</td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\sum Wz}{\sum W} = \frac{3.435}{2.153} = 1.595 \text{ in}, \quad \bar{z} = \frac{\sum Wz}{\sum W} = \frac{3.357}{2.153} = 1.599 \text{ in}.
\]

3. The truss is composed of equilateral triangles of sides \(a\) and is loaded and supported as shown. Determine the forces in members EF, DE, and DF.

4. Need reactions? Yes

Which one? E

Draw: E - D

\[
\sum F_x = 0: EF \sin 60^\circ = 2L/\sqrt{3} = 0
\]

\[
\sum F_y = 0: DF \cos 30^\circ - 2L/\sqrt{3} = 0
\]

\[
\sum M_r = 0: L(2a \cos 30^\circ) - R(3a \sin 30^\circ) = 0
\]

\[
R = 2L/\sqrt{3}
\]
4. **Question from exam** A Polynesian, or duopitch roof truss is loaded as shown.

(a) (5 points) What is the distance from point $A$ that the line of action of the resultant of the external loadings crosses the base of the truss?

**Solution:**

\[
R = 200 + 400 + 400 + 400 + 350 + 300 + 300 + 300 + 150 = 2800 \text{ lb} \downarrow
\]

\[
M_A = R \cdot d = 400 \cdot 6 + 400 \cdot 12 + 400 \cdot 18 + 350 \cdot 24 + 300 \cdot 30 + 300 \cdot 36 + 300 \cdot 42 + 150 \cdot 48 = 62,400 \text{ lb} \cdot \text{ft}
\]

\[\Rightarrow \quad d = 22.3 \text{ ft}\]

(b) (10 points) Determine the forces in members $FE$, $FH$, and $FG$.

**Solution:**

From the analysis of the whole truss:

**Free body: Truss:**

\[\Sigma F_x = 0: \quad N_x = 0\]

\[\Sigma M_A = 0: \quad (200 \text{ lb})(8a) + (400 \text{ lb})(7a + 6a + 5a) + (350 \text{ lb})(4a) + (300 \text{ lb})(3a + 2a + a) - A(8a) = 0\]

\[A = 1500 \text{ lb} \uparrow\]

We pass a section through $DF$, $EF$, and $EG$, and use the free body shown.

(We apply $F_{DF}$ at $F$.)

\[\Sigma M_A = 0: \quad F_{EF}(18 \text{ ft}) - (400 \text{ lb})(6 \text{ ft}) - (400 \text{ lb})(12 \text{ ft}) = 0\]

\[F_{EF} = +400 \text{ lb} \quad F_{EF} = 400 \text{ lb} \quad T \rightarrow\]
Now do a cut through $FH$, $FG$ and $EG$:

\[ \sum M_c = 0 = -A \cdot 24 + 200 \cdot 24 + 400 \cdot 18 + 400 \cdot 12 + 400 \cdot 6 - F_{FH} \cdot \frac{4}{\sqrt{4^2 + 6^2}} \cdot 6 - F_{FH} \cdot \frac{6}{\sqrt{4^2 + 6^2}} \cdot 4.5 \]

\[ \Rightarrow \quad F_{FH} = \frac{-16800}{7.07} = -2375.4 \text{ lb} \]

\[ \Rightarrow \quad F_{FH} = 2375 \text{ lb C} \]

\[ \sum F_y = 0 = A - 200 - 400 - 400 - 400 - F_{FH} \cdot \frac{4}{\sqrt{4^2 + 6^2}} - F_{FG} \cdot \frac{4.5}{\sqrt{4.5^2 + 6^2}} \]

\[ \Rightarrow \quad F_{FG} = \frac{-1217.4}{0.6} = -2029 \text{ lb} \]

\[ \Rightarrow \quad F_{FG} = 2029 \text{ lb C} \]

(e) (5 points) If the external force at point $B$ is removed, what the external force at $K$ needs to be in order for the forces in $AB$ and $AC$ to remain the same as in part (b)? No need to calculate the actual forces in the above members.

**Solution:** for $AB$ and $AC$ to remain the same, the reaction at $A$ needs to stay the same. In order for the reaction at $A$ to remain the same, the moment around $N$ created by the external loading needs to remain the same.

\[ B \cdot d_B + K \cdot d_K = const = 400 \cdot 42 + 300 \cdot 12 = K \cdot 12 \]
(d) (Bonus - 5 points) If the external force at point B is removed, by examination, what are the zero-force members? Explain.

**Solution:** joint B is a special case $\Rightarrow BC=0$. Now joint C becomes a special case, leading to $CD=0$.

5. **Question from midterm** A Pratt roof truss is loaded as shown in the figure to the right.

(a) (10 points) Neglect any horizontal reactions at the supports and solve for the forces in all the members if $P = Q = 1$ kN.

**Solution:**

By inspection of joint B (special case), $BA=BC=3.35$ kN C, $BH=P=1$ kN C.
(b) (5 points) If $P = 0$ kN, and $Q = 4$ kN, what are the forces in members $AB$ and $AH$? Explain.

**Solution:**

The symmetry no longer exists. However, equilibrium of moments around $E$ gives the reaction at $A$, which is the same as in part (a). Under these conditions, the analysis of joint $A$ remains the same, and the forces in $AB$ and $AH$ are the same as in (b).

(c) (5 points) Under the loading condition in part (b) identify by inspection those members in which the forces are zero. Explain.

**Solution:**

With load $P=0$, joint $B$ is a special case, giving $BH=0$. Given that, joints $H$ and $G$ are also special cases, giving $HC=GC=0$.