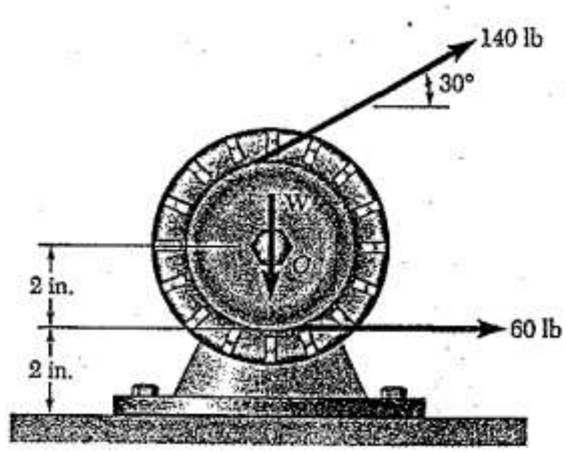
Honors engineering Statics – MECH 223h

Review Problems for Midterm 1

1. A 32-lb motor is mounted on the floor. Find the resultant of the weight and the forces exerted on the belt, and determine where the line of action of the resultant intersects the floor.



Solution:

FIRST REDUCE THE GIVEN FORCES TO AN EQUIVALENT FORCE-COUPLE SYSTEM AT O. THEN FOR EQUIVALENCE..

ΣF_x: $140\cos 30° + 60 = R_x$ OR $R_x = 181.244$ lb

ΣF_y: $140\sin 30° - 32 = R_y$ OR $R_y = 38$ lb

ΣM_O: $-(2\text{ in.})(140\text{ lb}) + (2\text{ in.})(60\text{ lb}) = M_O$

OR $M_O = -160$ lb·in

NEXT MOVE THE EQUIVALENT FORCE-COUPLE SYSTEM TO THE POINT P WHICH LIES ON THE FLOOR DIRECTLY BELOW O. THUS..

AT P.. $R_x = 181.244$ lb $\quad R_y = 38$ lb

AND ΣM_P: $-160\text{ lb·in} - (4\text{ in.})(181.244\text{ lb}) = M_P$

OR $M_P = -884.98$ lb·in.

FINALLY, REPLACE $(\underline{R}, \underline{M}_P)$ WITH THE SINGLE EQUIVALENT FORCE $\underline{R}$, WHERE..

$R = \sqrt{(181.244)^2 + (38)^2}$ $\quad \tan\theta = \frac{38}{181.244}$

$= 185.2$ lb

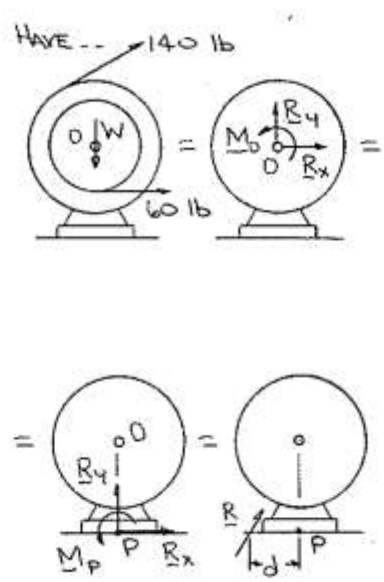
OR $\theta = 11.84°$

AND.. ΣM_P: $-884.98\text{ lb·in} = d\,(38\text{ lb})$

OR $d = 23.3$ in.

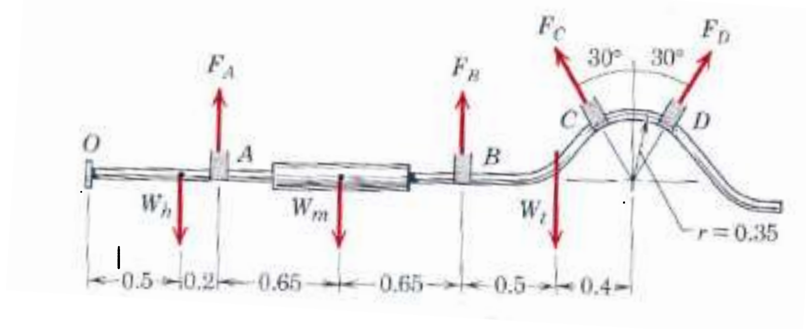
$\therefore \underline{R} = 185.2$ lb $\measuredangle 11.84°$ ◀

AND THE LINE OF ACTION OF $\underline{R}$ INTERSECTS THE FLOOR AT A POINT 23.3 IN. TO THE LEFT OF THE VERTICAL CENTER LINE OF THE MOTOR. ◀



2. An exhaust system for a pickup truck is shown in the drawing. The weights W_h, W_m, and W_t of the headpipe, muffler, and tailpipe are 10, 100, and 50 N respectively, and act at the indicated points. If the exhaust hanger at point *A* is adjusted so that its tension F_A is 50 N, determine the required forces in the hangers at points *B, C,* and *D* so that the force couple system at point O is zero.

Dimensions in Meters



Solution:

For a zero force - couple system at point O:

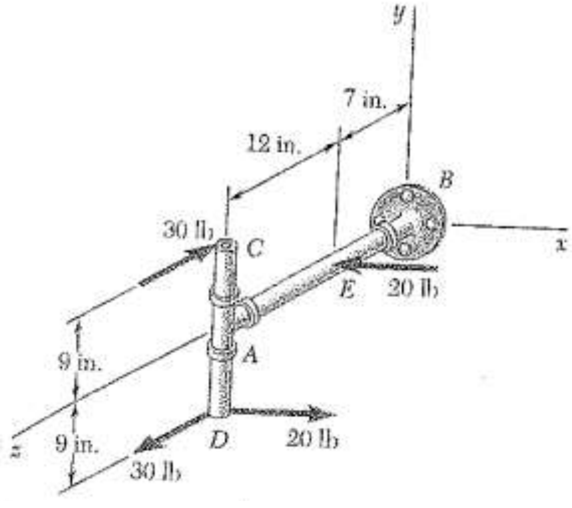
$$\underline{R} = \Sigma \underline{F} = (-F_C \sin 30° + F_D \sin 30°)\,\underline{i} + (50 - 10 - 100 - 50 + F_B + F_C \cos 30° + F_D \cos 30°)\,\underline{j} = \underline{0}$$

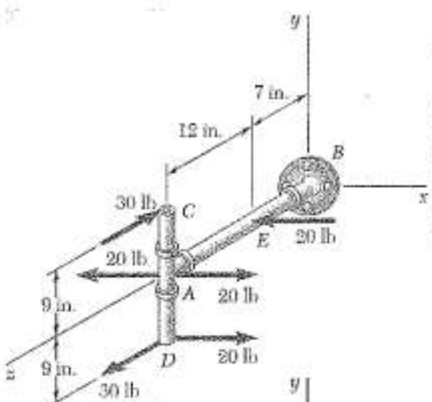
$$\Rightarrow F_C = F_D = F$$

$$\circlearrowleft M_o = -10(0.5) + 50(0.7) - 100(1.35) + F_B(2) - 50(2.5) + 2F \cos 30° (2.9) = 0$$

$$\underline{F = F_C = F_D = 6.42 \text{ N}}, \quad \underline{F_B = 98.9 \text{ N}}$$

3. Determine the components of a single couple equivalent to the two couples shown

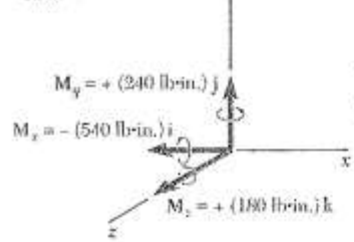




SOLUTION

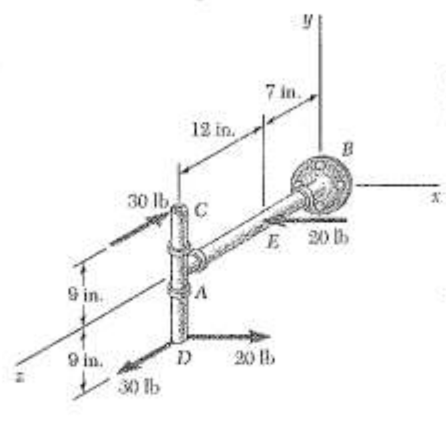
Our computations will be simplified if we attach two equal and opposite 20-lb forces at A. This enables us to replace the original 20-lb-force couple by two new 20-lb-force couples, one of which lies in the zx plane and the other in a plane parallel to the xy plane. The three couples shown in the adjoining sketch can be represented by three couple vectors $\mathbf{M}_x$, $\mathbf{M}_y$, and $\mathbf{M}_z$ directed along the coordinate axes. The corresponding moments are

$$M_x = -(30 \text{ lb})(18 \text{ in.}) = -540 \text{ lb} \cdot \text{in.}$$
$$M_y = +(20 \text{ lb})(12 \text{ in.}) = +240 \text{ lb} \cdot \text{in.}$$
$$M_z = +(20 \text{ lb})(9 \text{ in.}) = +180 \text{ lb} \cdot \text{in.}$$



These three moments represent the components of the single couple **M** equivalent to the two given couples. We write

$$\mathbf{M} = -(540 \text{ lb} \cdot \text{in.})\mathbf{i} + (240 \text{ lb} \cdot \text{in.})\mathbf{j} + (180 \text{ lb} \cdot \text{in.})\mathbf{k} \quad ◀$$



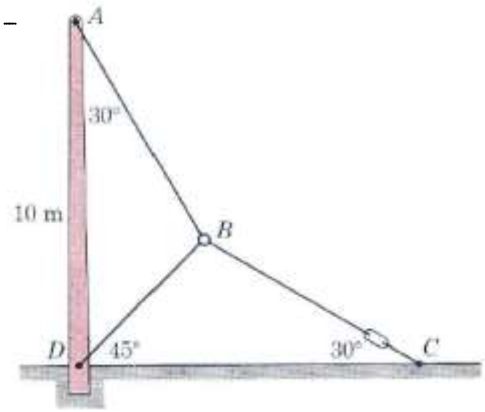
Alternative Solution. The components of the equivalent single couple **M** can also be obtained by computing the sum of the moments of the four given forces about an arbitrary point. Selecting point D, we write

$$\mathbf{M} = \mathbf{M}_D = (18 \text{ in.})\mathbf{j} \times (-30 \text{ lb})\mathbf{k} + [(9 \text{ in.})\mathbf{j} - (12 \text{ in.})\mathbf{k}] \times (-20 \text{ lb})\mathbf{i}$$

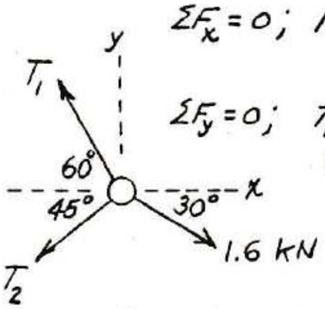
and, after computing the various vector products,

$$\mathbf{M} = -(540 \text{ lb} \cdot \text{in.})\mathbf{i} + (240 \text{ lb} \cdot \text{in.})\mathbf{j} + (180 \text{ lb} \cdot \text{in.})\mathbf{k} \quad ◀$$

4. The three cables in the drawing are secured to a ring at *B*, and the turnbuckle at *C* is tightened until it supports a tension of 1.6 kN. Calculate the moment *M* produced by the tension in cable *AB* about the base of the mast at *D*.



Solution:



$\Sigma F_x = 0;\quad 1.6\cos 30° - T_1\cos 60° - T_2\cos 45° = 0$

$0.5T_1 + 0.707T_2 = 1.386$

$\Sigma F_y = 0;\quad T_1\sin 60° - T_2\sin 45° - 1.6\sin 30° = 0$

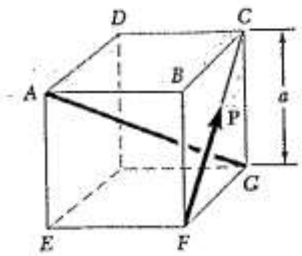
$0.866T_1 - 0.707T_2 = 0.8$

Add & get

$1.366T_1 = 2.186,\quad T_1 = 1.6 \text{ kN}$

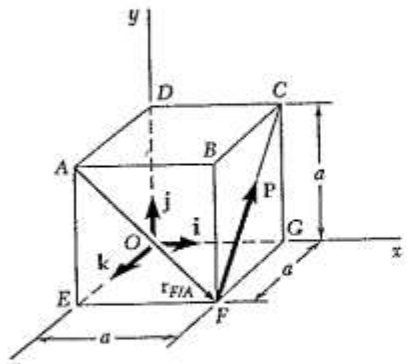
$(M_{T_1})_D = (1.6\sin 30°)10 = \underline{8 \text{ kN}\cdot\text{m}}$

5.



A cube of side a is acted upon by a force $\mathbf{P}$ as shown. Determine the moment of $\mathbf{P}$ (*a*) about A, (*b*) about the edge AB, (*c*) about the diagonal AG of the cube. (*d*) Using the result of part *c*, determine the perpendicular distance between AG and FC.

SOLUTION



***a.* Moment about *A*.** Choosing x, y, and z axes as shown, we resolve into rectangular components the force $\mathbf{P}$ and the vector $\mathbf{r}_{F/A} = \overrightarrow{AF}$ drawn from A to the point of application F of $\mathbf{P}$.

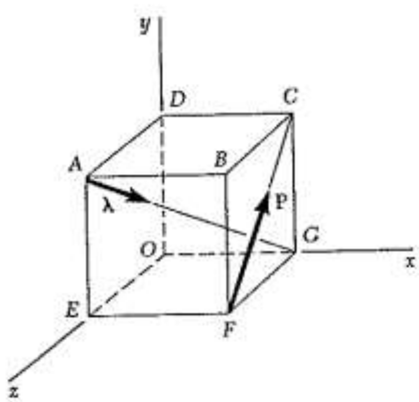
$$\mathbf{r}_{F/A} = a\mathbf{i} - a\mathbf{j} = a(\mathbf{i} - \mathbf{j})$$
$$\mathbf{P} = (P/\sqrt{2})\mathbf{j} - (P/\sqrt{2})\mathbf{k} = (P/\sqrt{2})(\mathbf{j} - \mathbf{k})$$

The moment of $\mathbf{P}$ about A is

$$\mathbf{M}_A = \mathbf{r}_{F/A} \times \mathbf{P} = a(\mathbf{i} - \mathbf{j}) \times (P/\sqrt{2})(\mathbf{j} - \mathbf{k})$$
$$\mathbf{M}_A = (aP/\sqrt{2})(\mathbf{i} + \mathbf{j} + \mathbf{k}) \quad ◀$$

***b.* Moment about *AB*.** Projecting $\mathbf{M}_A$ on AB, we write

$$M_{AB} = \mathbf{i} \cdot \mathbf{M}_A = \mathbf{i} \cdot (aP/\sqrt{2})(\mathbf{i} + \mathbf{j} + \mathbf{k})$$
$$M_{AB} = aP/\sqrt{2} \quad ◀$$



We verify that, since AB is parallel to the x axis, M_{AB} is also the x component of the moment $\mathbf{M}_A$.

***c.* Moment about Diagonal *AG*.** The moment of $\mathbf{P}$ about AG is obtained by projecting $\mathbf{M}_A$ on AG. Denoting by $\boldsymbol{\lambda}$ the unit vector along AG, we have

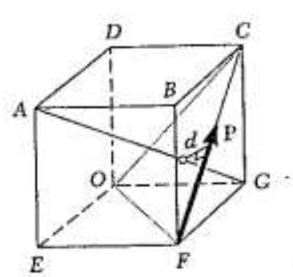
$$\boldsymbol{\lambda} = \frac{\overrightarrow{AG}}{AG} = \frac{a\mathbf{i} - a\mathbf{j} - a\mathbf{k}}{a\sqrt{3}} = (1/\sqrt{3})(\mathbf{i} - \mathbf{j} - \mathbf{k})$$
$$M_{AG} = \boldsymbol{\lambda} \cdot \mathbf{M}_A = (1/\sqrt{3})(\mathbf{i} - \mathbf{j} - \mathbf{k}) \cdot (aP/\sqrt{2})(\mathbf{i} + \mathbf{j} + \mathbf{k})$$
$$M_{AG} = (aP/\sqrt{6})(1 - 1 - 1) \qquad M_{AG} = -aP/\sqrt{6} \quad ◀$$

Alternative Method. The moment of $\mathbf{P}$ about AG can also be expressed in the form of a determinant:

$$M_{AG} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_{F/A} & y_{F/A} & z_{F/A} \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} 1/\sqrt{3} & -1/\sqrt{3} & -1/\sqrt{3} \\ a & -a & 0 \\ 0 & P/\sqrt{2} & -P/\sqrt{2} \end{vmatrix} = -aP/\sqrt{6}$$

***d.* Perpendicular Distance between *AG* and *FC*.** We first observe that $\mathbf{P}$ is perpendicular to the diagonal AG. This can be checked by forming the scalar product $\mathbf{P} \cdot \boldsymbol{\lambda}$ and verifying that it is zero:

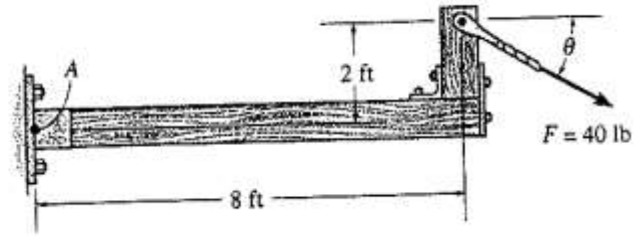
$$\mathbf{P} \cdot \boldsymbol{\lambda} = (P/\sqrt{2})(\mathbf{j} - \mathbf{k}) \cdot (1/\sqrt{3})(\mathbf{i} - \mathbf{j} - \mathbf{k}) = (P\sqrt{6})(0 - 1 + 1) = 0$$



The moment M_{AG} can then be expressed as $-Pd$, where d is the perpendicular distance from AG to FC. (The negative sign is used since the rotation imparted to the cube by $\mathbf{P}$ appears as clockwise to an observer at G.) Recalling the value found for M_{AG} in part *c*,

$$M_{AG} = -Pd = -aP/\sqrt{6} \qquad d = a/\sqrt{6} \quad ◀$$

6. Determine the direction θ (0°≤θ≤90°) of the force F=40 lb so that it produces (a) maximum moment about A and (b) minimum moment about A. Compute the moment in each case.



Solution:

We can break the force F into components $\bar{F} = F_x\bar{\imath} + F_y\bar{\jmath} = F(\cos\theta\,\bar{\imath} - \sin\theta\,\bar{\jmath})$. The total moment about A will be the sum of moments created by each of the components. Both components create a clockwise moment (negative). Using the 2D scalar definition (M=dF) we get:

$$M(\theta) = -F(2\,ft * \cos\theta + 8\,ft * \sin\theta)$$

(a) To find the maximum, take the derivative with respect to θ and equate to 0:

$$-\frac{M'(\theta)}{F} = -2\,ft * \sin\theta + 8\,ft * \cos\theta = 0$$

$$\Rightarrow\ \tan\theta = 4$$

$$\boxed{\Rightarrow\ \theta = 75.96°}$$

$$M(\theta = 75.96°) = -40\,lb * (2\,ft * \cos 75.96° + 8\,ft * \sin 75.96°) =$$
$$-(19.40 + 310.45)lb * ft = -330\,lb * ft$$

$$\boxed{M(\theta = 75.96°) = -330\,lb * ft}$$

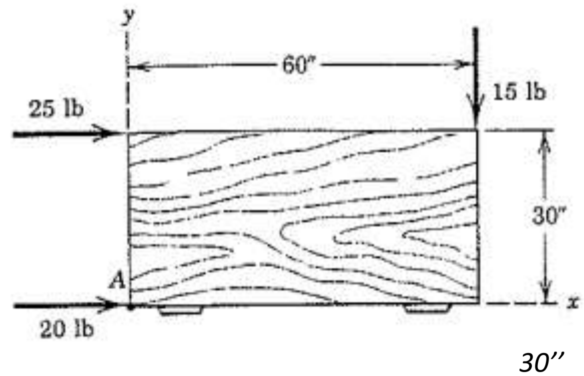
Note: This moment is larger than the one for θ=90°, which would be -320 lb*ft.

(b) The minimum moment in the range of the given angles happens when $\boxed{\theta=0°}$.

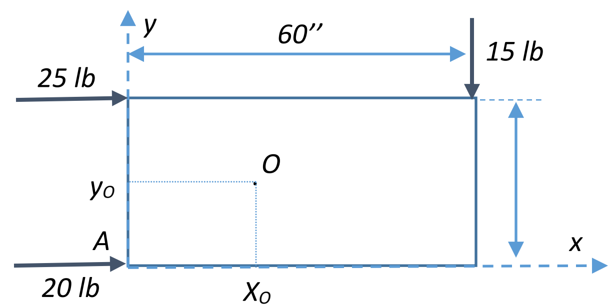
In this case the force is reduced to its horizontal component, and the corresponding moment is:

$$\boxed{M(\theta = 0°) = -40\,lb * 2ft = -80\,lb * ft}$$

7. While sliding a desk toward the doorway, three students exert the forces shown in the overhead view. Determine the equation of the line of action of the resultant force.



Solution:



First, replace the given system of forces with an equivalent force couple system at A:

$$\bar{R} = (45)lb\,\bar{\imath} - 15\;lb\,\bar{\jmath}$$

$$M = \sum M_{Ai} = \sum F_i d_i =$$

$$= 25\;lb * 30'' + 15\;lb * 60'' = 1650\;lb * inch\;\circlearrowright$$

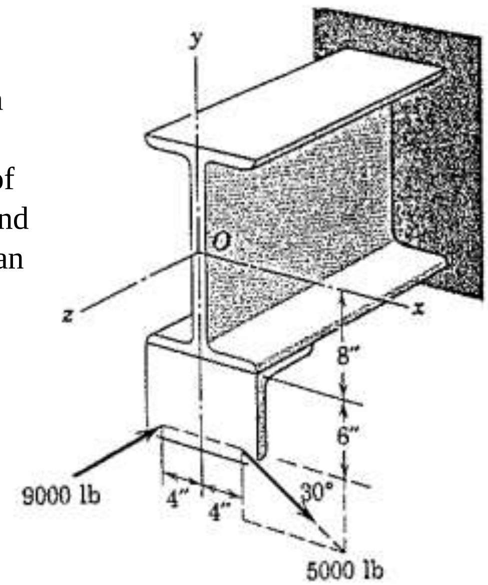
Now shift the line of action of the resultant to point O where the moment around A is equal to the couple moment we just calculated. The moment of the resultant at O around A is the sum of the moments of the components of $\bar{R}$.

$$M = 1650\;lb * inch = 45\;lb * y_O + 15\;lb * x_O$$

Rearranging the equation gives us the formula for the line of action of $\bar{R}$.

$$\boxed{y_O = -\frac{15}{45}x_O + \frac{1650}{45} = -\frac{1}{3}x_O + \frac{110}{3}}$$

8. A right-angle bracket is welded to the flange of the I-beam to support the 9000-lb force applied parallel to the axis of the beam, and the 5000-lb force, applied in the end plane of the beam. In analyzing the capacity of the beam to withstand the applied loads, it is convenient to replace the forces by an equivalent force at *O* and a corresponding couple **M**. Determine the *x*-,*y*-, and *z*- components of **M**.

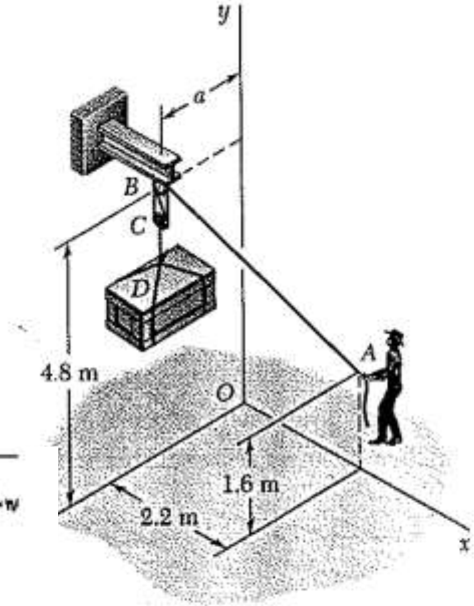


Solution:

The components of **M** are the moments around the corresponding axes. The 9000-lb force creates a moment around the y axis and around the x axis (no moment around the z axis from this force because the force is parallel to the axis). If we call the 5000-lb force **Q**, then Q_x=5000*Cos30° lb=4330 lb and Q_y=-5000*Sin30° lb=-2500 lb. Since both Q_x and Q_y are in the xy plane, they create only moments around the z axis.

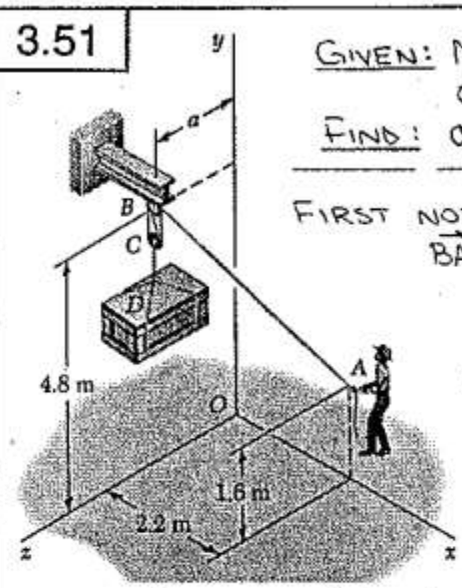
$$\Rightarrow \quad M_x = 9000\ lb * 14'' = 126{,}000\ lb * inch\ \circlearrowleft = 10{,}500\ lb * foot\ \circlearrowleft$$
$$M_y = 9000\ lb * 4'' = 36{,}000\ lb * inch\ \circlearrowright = 3000\ lb * foot\ \circlearrowright$$
$$M_z = 4330\ lb * 14'' - 2500\ lb * 4'' = 50{,}620\ lb * inch\ \circlearrowleft \approx 4220\ lb * foot\ \circlearrowleft$$

9. To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook *B***.** Knowing that the moments about the *y* and the *z* axes of the force exerted at *B* by portion *AB* of the rope are, respectively 120 N·m and -460 N·m, determine the distance *a*.



Solution:

3.51



GIVEN: $M_y = 120$ N·m, $M_z = -460$ N·m OF $\underline{T}_{BA}$ AT B

FIND: a

FIRST NOTE..

$$\overrightarrow{BA} = (2.2\text{ m})\underline{i} - (3.2\text{ m})\underline{j} - (a\text{ m})\underline{k}$$

NOW.. $\underline{M}_O = \underline{r}_{A/O} \times \underline{T}_{BA}$

WHERE

$$\underline{r}_{A/O} = (2.2\text{ m})\underline{i} + (1.6\text{ m})\underline{j}$$

$$\underline{T}_{BA} = \frac{T_{BA}}{d_{BA}}(2.2\,\underline{i} - 3.2\,\underline{j} - a\,\underline{k})\ (\text{N})$$

THEN..

$$\underline{M}_O = \frac{T_{BA}}{d_{BA}}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2.2 & 1.6 & 0 \\ 2.2 & -3.2 & -a \end{vmatrix}$$

$$= \frac{T_{BA}}{d_{BA}}\left\{-1.6a\,\underline{i} + 2.2a\,\underline{j} + [(2.2)(-3.2) - (1.6)(2.2)]\underline{k}\right\}$$

THUS.. $M_y = 2.2\dfrac{T_{BA}}{d_{BA}}a$ (N·m)

$$M_z = -10.56\frac{T_{BA}}{d_{BA}}\ (\text{N·m})$$

THEN.. FORMING THE RATIO $\dfrac{M_y}{M_z}$ --

$$\frac{120\text{ N·m}}{-460\text{ N·m}} = \frac{2.2\frac{T_{BA}}{d_{BA}}a\ (\text{N·m})}{-10.56\frac{T_{BA}}{d_{BA}}\ (\text{N·m})}$$

OR $a = 1.252$ m